Reg. No .... Name **24U328**

B.Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024

SEMESTER 3 : COMPUTER APPLICATIONS

COURSE : **19U3CRCMT3 : CALCULUS**

(For Regular 2023 Admission and Improvement/Supplementary 2022/2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Expand ln sin x in powers of (x-2).
2. Find the volume and surface area of the torus (doughnut) generated by revolving a circular disc of radius a about an axis in its plane at a distance b≥a from its center.
3. Evaluate the integral $\int_{\pi/6}^{\pi/3}\left(1-\cos\ 3t\right)\sin\ 3t\ dt.$
4. Evaluate $\int_0^1\int_0^2 x\ y\left(x-y\right)\ \mathrm{d}\,x\ dy.$
5. If f(x,y)=x $\tan^{-1}$(xy), find $\frac{\partial f}{\partial x}\ and\ \frac{\partial f}{\partial y}.$
6. Evaluate $\int_0^\infty\int_x^\infty \frac{e^y}{y}\ dy\ dx$.
7. Compute the second order partial derivatives of the function $f\left(x,y\right)=x^2y+\cos\ y+y\ \sin\ x.$
8. Evaluate $\int_{-1}^0\int_{-1}^1\left(x\ +y+1\right)\ dx\ dy.$
9. $Find\ the\ n^{th}\ derivative\ of\ \cos^3\ x.$
10. Find f_x ,f_y and f_z if f(x,y,z)= ln(x+2y+3z).
11. Find the area between the curves y= $\sec^2$ x and y= sin x from 0 to $\pi/4.$
12. Find the points of inflection of the curve $y=3x^4-4x^3+1.$

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. a) Solve the system u = x - y and v = 2x + y for x and y in terms of u and v. Then find the value of the Jacobian J(u, v).
    b) Find the image under transformation u = x - y and v = 2x + y of the triangular region with vertices (0, 0), (1, 1) and (1, -2) in the xy-plane. Sketch the transformed region in the uv-plane.
14. If $y=\left(\sin^{-1}x\right)^2,\ prove\ that\ (1-x)^2y_{n+2}-\left(2n+1\right)x\ y_{n+1}-n^2y_n=0.$
15. Find the volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines y = x and x = 1and whose top lies in the plane z = f(x, y)= 3 - x - y.
16. Show that the evolute of the asteroid $x=a\ \cos^3 t,\quad y=a\ \sin^3 t$ is $(x+y)^{2/3}+(x-y)^{2/3}=2\ a^{2/3}.$

17. Use shell method, to find the volume of the solid generated by revolving the region bounded by the lines y = 2, y =-x/2, x = 2 about y-axis.

18. A rectangular box, open at the top,is to have a volume of 32 cubic feet.What must be the dimensions so that the total surface is a minimum.

19. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line x=3 about the line x=3.

20. Using chain rule express $\frac{\partial w}{\partial r} and\ \frac{\partial w}{\partial \theta}$ in terms of $r\ and\ \theta$, if $w = \tan^{-1}\left(y/x\right),\ x = r\ \cos\theta,\ y = r\ \sin\theta.$ Also evaluate $\frac{\partial w}{\partial r} and\ \frac{\partial w}{\partial \theta}$ at the point (1, π/6).

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Using Lagrange multipliers, find the maximum and minimum values of 3x-y+6 subject to the constraint $x^2 + y^2 = 4.$

22. a) Find the area of the surface generated by revolving about the axis of x, the arc of the parabola $y^2 = 4ax$ from the origin to the point where x = a, a>0.
b)The region bounded by the curve $y = x^2 + 1$ and the line y = x + 3 is revolved about the x-axis to generate a solid. Find the volume of the solid by washer method.

23. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by applying the transformation u = x/a, v = y/b, w = z/c.

24.
$$If\ \ y = \sin\left(m\ \sin^{-1} x\right),\ show\ that$$

$$y_n\left(0\right) = \begin{cases} 0, & if\ n\ is\ even \\ m\left(1 - m^2\right)\left(3^2 - m^2\right)\ldots\ldots\ldots\left[\left(n-2\right)^2 - m^2\right], & if\ n\ is\ odd \end{cases}$$

(10 x 3 = 30)