Reg. No .... Name **24U327**

B.Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024

SEMESTER 3 : MATHEMATICS

COURSE : **19U3CRMAT3 : VECTOR CALCULUS, THEORY OF EQUATIONS AND MATRICES**

(For Regular 2023 Admission and Improvement/Supplementary 2022/2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Find $\iint_s \bar{F} \,.\, \hat{n} ds$, Where $\bar{F}$ = $\left(2x+3z\right)i-\left(xz+y\right)j+\left(y^2+2z\right)k$ and s is the surfsce of the sphere of the sphere having center at (3,-1,2) and radius 3.
2. If $\bar{r} = ti - t^2 j + (t-1)k \; and \; \bar{s} = 2t^2 i + 6kt, evaluate \; \int_0^2 \bar{r} \times \bar{s} dt.$
3. Any real polynomial equation of odd degree has at least one real root. Justify.
4. Define homogeneous equation and give the condition for trivial solution and non trivial solution.
5. Find the angle between the surface $x^2+y^2+z^2 \; = 9 \; and \; z = x^2+y^2-z \; at \; the \; point \; \left(2,-1,2\right).$
6. State the consistancy of a system of equation.
7. Find the unit vector normal to the surface $i)xy^3z^2 = 4 \; at \; \left(-1,-1,2\right)$

   $ii)x^2y+2xz = 4 \; at \; \left(2,-2,3\right)$
8. Find a unit vector normal to the surface $x^3+y^3+3xyz = 3$ at the point (1,2,-1).
9. Evaluate the eigen values of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.
10. State Descarte's rule of signs and apply it to prove that the equation $x^3+2x+3=0$ has one negative and two imaginary roots.
11. Explain the term normal form of a matrix with examples.
12. If α, β, γ are the roots of the equation $x^3-px^2+qx-r=0$, find the value of $\sum \alpha^2$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Use divergence theorem to evaluate $\iint_s \overline{F} \,.\, ds$ where $\overline{F} = x^3 i + y^3 j + z^3 k$ and S is the surface of the sphere $x^2+y^2+z^2 = a^2$.
14. Find the directional derivative of the function $f\left(x,y,z\right) = xy^2+yz^3 \; at \; the \; point \; \left(2,-1,1\right)$ in the direction of $i+2j+2k$.
15. If $\bar{r} = xi + yi + zk \; show \; that \; \nabla r^n = nr^{n-2}\bar{r}.$
16. Reduce to normal form to evaluate the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.

17. Evaluate the rank of the matrix $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$.

18. The acceleration of a particle at time t is given by $\bar{a} = 18\ \cos\ 3ti - 8\sin 2tj + 6tk$.If velocity $\bar{v}$ and displacement $\bar{r}$ be zero at t=0,Find $\bar{v}\ and\ \bar{r}$.

19. Solve the equation $x^4+5x^3-30x^2+40x+64=0$ whose roots are in G.P

20. Solve $8x^3 - 47x^2 + 66x + 9 = 0$ given that it has a double root.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Verify if the vector field $\overline{A} = 2xyz^3i + x^2z^3j + 3x^2yz^2k$. is a) solenoidal b) irrotational c)find its scalar potential .

22. Calculate the eigen values and the eigen vectors of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

23. Verify Stoke's theorem for the vector field $\overline{F} = (2x - y)i - yz^2j - y^2zk$ over the upper half surface of $x^2 + y^2 + z^2 = 1$,bounded by its projection on the xy -plane.

24. (a) Solve $x^4 + 12x^2 + 8x + 6 = 0$ using Ferrari's method
(b) Solve $x^3 - 6x^2 + 3x - 2 = 0$ using Cardan's method.

(10 x 3 = 30)