Reg. No .... Name **24P322**

M. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT12 : FUNCTIONAL ANALYSIS**

(For Regular 2023 Admission and Supplementary 2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	Let X and Y be normed linear spaces. If X is finite dimensional , show that every linear map $F : X \to Y$ is continuous.	(A, CO 1)
2.	Explain the completion X_c of a nls X.	(U, CO 4)
3.	Show that every finite dimensional nls is a Banach space.	(A, CO 2)
4.	Explain the concept of normed linear space. Give an example.	(U, CO 1)
5.	Define Schauder basis for a nls X.	(R, CO 2)
6.	Let X be a Banach space and $A \in BL(X)$. Define eigen value of A, eigen vector corresponding to an eigen value and eigen space corresponding to an eigen value.	(R, CO 3)
7.	Let X be a Banach space. Is $BL(X)$ is a Banach space? Justify your answer.	(An, CO 3)
8.	Let X and Y be normed linear spaces. For $F \in BL(X, Y)$, define $\|\|F\|\| = sup\{\|\|F(x)\|\| : x \in X, \|\|x\|\| \leq 1\}$.Show that $\|\|F(x)\|\| \leq \|\|F\|\|.\|\|x\|\|, \forall x \in X$.	(An, CO 1)
9.	Let X be a Banach space and $A \in BL(X)$. Show that the map defined on the set of all invertible elements in $BL(X)$, which takes an invertible element $A \in BL(X)$ to its inverse A^{-1} in $BL(X)$, is continuous.	(An, CO 3)
10.	Let X and Y be normed linear spaces and $F : X \to Y$ be a linear map. Show that F is continuous at zero if and only if F is continuous on X.	(U, CO 1)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	State and prove the Banach - Steinhaus theorem.	(E, CO 2)
12.	Let X and Y be Banach function spaces and let x_0 be a K-valued function on T such that $x_0 x \in Y$ whenever x is an element in X. Show that the map $F : X \to Y$ defined by $F(x) = x_0 x, \forall x \in X$ is continuous.	(An, CO 2)
13.	Let X be a Banach space. Give an example of an operator $A \in BL(X)$, which has a spectral value that is not an eigen value; in other words, give an example of $A \in BL(X)$, such that $e(A)$ is a proper subset of $s(A)$.	(An, CO 3)
14.	Let X be a normed linear space and $a \neq 0 \in X$.Show that there is an $f \in X'$ such that $f(a) = \|\|a\|\|$ and $\|\|f\|\| = 1$.	(An, CO 1)
15.	Let X be a normed linear space. If Y is a subspace of X, prove that $Y \neq X$ if and only if $Y^\circ = \emptyset$.	(An, CO 1)
16.	Construct a linear functional which is not continuous.	(An, CO 1)
17.	Let X be a normed linear space. Show that there exists a unique Banach space X_c and a linear isometry F_c of X into X_c such that $F_c(X)$ is dense in X_c.	(An, CO 4)

18. Let X be a Banach space and $A \in BL(X)$. Prove that the spectrum of A, $s(A)$ is a compact subset of K (An, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. State and prove the closed graph theorem. (An, CO 2)

20. State and prove the Hahn-Banach extension theorem. (E, CO 1)

21. Let A be invertible in $BL(X)$. If $||B - A|| < \frac{1}{||A^{-1}||}$, prove that B is invertible and $||B^{-1} - A^{-1}|| \leq \frac{||A^{-1}||^2||B-A||}{1-\epsilon}$, where $\epsilon = ||A^{-1}||||A - B||$. (An, CO 3)

22. Let X and Y be Banach spaces and $F \in BL(X, Y)$. Show that F is onto if and only if F' is bounded below, that is if and only if $\alpha||y'|| \leq ||F(y')||$ for every $y' \in Y'$ and for some $\alpha > 0$ (E, CO 4)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Analyze normed linear spaces, continuity of linear maps , theory and applications of the Hahn-Banach Theorems	E	1, 4, 8, 10, 14, 15, 16, 20	15
CO 2	Analyze Banach spaces, Uniform boundedness principle and the Closed graph theorem.	E	3, 5, 11, 12, 19	11
CO 3	Analyze the Open Mapping theorem, the eigen spectrum and spectral radius.	E	6, 7, 9, 13, 18, 21	12
CO 4	Analyze duals of a normed linear space and transposes of bounded linear maps.	E	2, 17, 22	8

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;