

B.Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024

SEMESTER 5 : COMPUTER APPLICATION

COURSE : 19U5CRCMT6 : MATHEMATICAL ANALYSIS

(For Regular 2022 Admission and Supplementary 2021/ 2020 / 2019 Admissions)

Time : Three Hours

Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. a) Give an example of a bounded set having infinite number of limitpoints.
b) Give an example of an unbounded set with limit points.
2. Reduce $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ into a real number.
3. Show that the set $S = \{x/0 < x < 1, x \in R\}$ is open but not closed.
4. (a) Define upperbound of a set S
(b) Find the supremum of the set $\{\frac{1}{n} : n \in \mathbb{N}\}$
5. Represent the complex number $z = -4 + i4\sqrt{3}i$ in exponential form.
6. Find the supremum and infimum of the set $\{1,3,5,7,9\}$?
7. Find the principal argument of $-21+3i$.
8. Show that $\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{n^2} = \frac{1}{2}$.
9. State the equivalence of two forms of Completeness property of real numbers.
10. a) Define monotonic sequence with an example.
b) Define Cauchy sequence.
11. Find the limit inferior and limit superior of the sequence $\{1+(-1)^n ; n \in \mathbb{N}\}$
12. Show that every open interval is an open set.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Prove that closure of every set is closed.
14. Show that $\lim_{n \rightarrow \infty} n^{1/n} = 1$.
15. Find all roots in rectangular coordinates of $(-16)^{1/4}$.
16. Prove: The union of two closed sets is closed.
17. Find all roots in rectangular coordinates of $(-8 - 8\sqrt{3}i)^{1/4}$.
18. Show that $\lim_{n \rightarrow \infty} \left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}} \right] = 1$.
19. Let $A, B \subseteq \mathbf{R}$ such that $A \subseteq B$. Show that $\text{Sup } A \leq \text{Sup } B$
20. Show that if \mathbf{R} has Dedekind's property, then it is order complete.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. If a sequence of closed intervals $[a_n, b_n]$ is such that each member $[a_{n+1}, b_{n+1}]$ is contained in the preceding one $[a_n, b_n]$ and $\lim(b_n - a_n) = 0$, then prove that there is one and only one point in common to all the intervals of the sequence.
22. State and prove Bolzano- Weierstrass Theorem for sets. Is its converse true? Justify.
23. State and prove Cauchy's general principle of convergence.
24. Show that the set of rational numbers is not order-complete.

(10 x 3 = 30)