Reg. No .... Name **24U540**

B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024

SEMESTER 5 : MATHEMATICS

COURSE : **19U5CRMAT7: ALGEBRA**

For Regular 2022 Admission and Supplementary 2021/ 2020 / 2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Define evaluation homomorphism.
2. Show that the factor group of an abelian group is abelian.
3. Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$, and let C be the subring of F consisting of all the constant functions in F. Is C an ideal in F? Why?
4. Find the orbits of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$.
5. A cyclic group has a unique generator. True or false. Justify
6. Define ring with unity.
7. Verify whether $a * b = a/b, \quad \forall a, b \in \mathbb{Q}$ is a binary operation on $\mathbb{Q}$.
8. Find the product $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$.
9. Define left coset of a subgroup.
10. Define simple group.
11. Give an example of binary operation on the set of irrational numbers.
12. Define conjugation of x by g.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Prove that the evaluation homomorphism can be defined on the group of all functions on real numbers.
14. Prove that direct product of the groups is a group.
15. Show that the group $\prod_{i=1}^{n} \mathbb{Z}_{m_i}$ is cyclic if and only if the numbers m_i are relatively primes in pairs.
16. If G is a group, then show that the identity and the inverses are unique.
17. Draw the subgroup diagram of Klein 4 group.
18. Show that a ring homomorphism $\phi : R \to R'$ is a one to one map if and only if Ker(ϕ)$= \{0\}$.
19. Show that the left cosets of a normal subgroup form a group under coset multiplication.
20. Prove that a field F is either of prime characteristic p and contains a subfield isomorphic to $\mathbb{Z}_p$ or of characteristic 0 and contains a subfield isomorphic to $\mathbb{Q}$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Show that if a and m are relatively prime integers, then for any integer b, then $ax \equiv b(\mod m)$ has as solutions of all integers in precisely one residue class modulo m. Find all solutions of $12x \equiv 27(\mod 18)$.

22. Compute $(\mathbb{Z}_4 \times \mathbb{Z}_6)/\langle(2,3)\rangle$.

23. Prove that every permutation in S_n can be written as a product of at most $n-1$ transpositions.

24. Prove or disprove: $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{R}^* \right\}$ is a group under matrix multiplication, where $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$

(10 x 3 = 30)