Reg. No .... Name **24P337**

M.Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024

SEMESTER 3 : MATHEMATICS

COURSE : 21P3MATT13 : ADVANCED TOPOLOGY

(For Regular 2023 Admission and Supplementary 2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Let A be a subset of X and let $x \in X$. If $x \in \bar{A}$, then prove that there exists a net in A which converges to x. ()
2. Is $\mathbb{R}$ locally compact? Justify. ()
3. Prove that T_0- axiom is a productive property. ()
4. Define the net associated with a filter on X. ()
5. If a topological space is embeddedable into a cube, prove that the space is Tychonoff. ()
6. Prove that T_1-axiom is a productive property. ()
7. Prove that the evaluation function of a family of functions is one-one iff that family distinguishes points. ()
8. Define a locally compact space and give an example of a locally compact space which is not compact. ()
9. State Urysohn's Lemma. (An)
10. Define an ultra filter on a set X. ()

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Prove that every continuous real- valued function on a countably compact space is bounded and attains its extreme. ()
12. Prove that a space X is connected if and only if each co-ordinate space is so.. (U)
13. If X is a Tychonoff space, prove that the family of all continuous real valued functions on X distinguishes points. ()
14. State Urysohn's characterization of normality and using it prove that a connected, T_4 space with at least two points must be uncountable. ()
15. Define co-final subset of a directed set. Suppose $S : D \to X$ is a net and F is a co-final subset of S. If $S/F : F \to X$ converges to a point x in X, then prove that x is a cluster point of S. ()
16. If a space is second countable and T_3, prove that it is embeddable in the Hilbert cube. ()
17. Let $S : D \to X$ be a net in a space X and let $x \in X$. Then Prove that x is a cluster point of S iff there exists a subnet of S which converge to x in X. ()
18. If a space X is regular and locally compact at a point $x \in X$, then prove that x has a local base consisting of compact neighbourhoods. ()

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Prove that a topological product is T_0, T_1, T_2 or regular iff each co-ordinate space has the corresponding property ()

20. State and prove Urysohn's Metrisation theorem. ()

21. For a topological space X, prove that the following statements are equivalent. ()

    a. X is compact
    b. Every net in X has a cluster point in X.
    c. Every net in X has a convergent subnet in X.

22. Prove that a subspace of a locally compact, Hausdorff space is locally compact iff it is open in its closure. ()

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;