

B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024**SEMESTER 5 : MATHEMATICS****COURSE : 19U5CRMAT06 : DIFFERENTIAL EQUATIONS***(For Regular 2022 Admission and Supplementary 2021/ 2020 / 2019 Admissions)*

Time : Three Hours

Max. Marks: 75

PART A**Answer any 10 (2 marks each)**

1. Find the complementary function of the equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x^2$
2. Write a Lagrange's differential equation.
3. What is the unique solution of $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + 4x\frac{y}{x} + x^2y=0$ such that $f(2) = f'(2) = f''(2) = 0$
4. Write the Bessel's equation.
5. If p and q are the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$, then what is the value of $2p - 3q$?
6. What is the standard solution of a linear differential equation?
7. What is the auxiliary equation of the Lagrange's linear partial differential equation?
8. Form a linear second order homogeneous differential equation whose solution is $y = c_1e^{-2x} + c_2e^{3x}$.
9. Find the singular points of $(x - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \frac{1}{x}y = 0$
10. Find the integrating factor of the linear equation $(x^2 + 1)\frac{dy}{dx} + 4xy = x$.
11. Form a partial differential equation by eliminating arbitrary constants from $z = ax + by + a^2 + b^2$
12. Write the normalised form of the differential equation $2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x - 5)y = 0$

(2 x 10 = 20)**PART B****Answer any 5 (5 marks each)**

13. Find the solution of the equation $(x^2 - y^2)dx = 2xydy$
14. Show that $\frac{d}{dx}[x^p J_p(kx)] = kx^p J_{p-1}(kx)$
15. Use the operator method to solve the following system of equations $\frac{dx}{dt} + \frac{dy}{dt} - 2x - 2y = e^t$, $\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$
16. Solve $\frac{d^3y}{dx^3} + \frac{dy}{dx} = 2x^2 + 4 \sin x$
17. Find the general integral of $x(y - z)p + y(z - x)q = z(x - y)$

18. Determine whether the following equation is exact .If it is exact, then solve it.
 $(3x^2y + e^y)dx + (x^3 + xe^y - 2y)dy = 0$
19. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + 4e^{3x}$.
20. Find the integral curves of $\frac{dx}{x^2 + 2y^2} = \frac{dy}{-xy} = \frac{dz}{xz}$

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Find the power series solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$
22. Solve $(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$.
23. Use the method of variation of parameters to solve
 $(x^2 + 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 6(x^2 + 1)^2$,
if the complementary function is $c_1x + c_2(x^2 - 1)$
24. (a) Solve the partial differential equation $z(xp - yq) = y^2 - x^2$
(b) Find a partial differential equation of all spheres whose centers lie on the z axis.

(10 x 3 = 30)