Reg. No .... Name **24U504**

B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2024

SEMESTER 5 : MATHEMATICS

COURSE : **19U5CRMAT05 : REAL ANALYSIS - I**

For Regular 2022 Admission and Supplementary 2021/ 2020 / 2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. If S and T are subsets of real numbers, then prove that $S \subseteq T \Rightarrow S' \subseteq T'$.
2. Find the infimum and supremum of the set $\{-2, -\frac{3}{2}, -\frac{4}{3}, -\frac{5}{4}, \ldots, -\frac{n+1}{n}, \ldots\}$. Which of these belongs to the set?
3. State Raabe's test for convergence of a series.
4. Give an example of a subset of $\mathbb{R}$ which is not order complete.
5. Define a monotonic increasing sequence and give an example.
6. Define dense set. Give an example.
7. Explain the concept of convergence of a series $\sum u_n$.
8. Evaluate $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$.
9. Define limit point of a sequence. What are the limit points of the sequence $\{\frac{1}{n}\}$?
10. Define a bounded below sequence and give an example.
11. Define limit inferior of a sequence.
12. Show that the series $\sum \dfrac{(n+1)^{n^2}}{3^n n^{n^2}}$ is convergent.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Test the convergence of the series $\sum \dfrac{1}{n^{1+1/n}}$.
14. Test the convergence of the series $x + 2x^2 + 3x^3 + 4x^4 + \cdots$
15. Show that the intersection of a finite collection of open sets is open. Is this theorem valid for an arbitrary family of open sets? Justify.
16. If $x, y \in \mathbb{R}$, prove that,
    (a) $|xy| = |x||y|$
    (b) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$, provided $y \neq 0$.
17. Show that the sequence $\{S_n\}$ where $S_n = \dfrac{1}{n+1} + \dfrac{1}{n+2} + \ldots + \dfrac{1}{n+n}, \quad \forall n \in \mathbb{N}$ is convergent.

18. Test for convergence of the series $\sum \frac{(n^3+1)^{1/3} - n}{\log n}$

19. If f and g are two functions defined on some neighbourhood of a point of c such that $\lim_{x \to c} f(x) = l$ and $\lim_{x \to c} g(x) = m$, prove that $\lim_{x \to c} (f - g)(x) = \lim_{x \to c} f(x) - \lim_{x \to c} g(x) = l - m$

20. Prove that a monotonic increasing sequence which is not bounded above diverges to $+\infty$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Show that a set is closed if and only if its complement is open.
22. State and prove D'Alembert's ratio test.
23. Show that a sequence is convergent if and only if it is bounded and has a unique limit point.
24. State and prove the logarithmic test for positive term series.

(10 x 3 = 30)