

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024
SEMESTER 4 - COMPLEMENTARY MATHEMATICS FOR PHYSICS AND CHEMISTRY
COURSE : 19U4CPMAT04 - FOURIER SERIES, LAPLACE TRANSFORMS, FOURIER
TRANSFORMS, AND GROUPS

(For Regular - 2022 Admission and Improvement / Supplementary - 2021/2020/2019 Admissions)

Time : Three Hours

Max. Marks: 75

PART A**Answer any 10 (2 marks each)**

1. Write the inversion Formula of the Fourier Sine Transform.
2. Find the Laplace transform of $e^{at} \sin bt$
3. Define Laplace transform and find the Laplace transform of $\cos ax$
4. Write the inversion Formula of the complex Fourier Transform.
5. Is the binary operation on \mathbb{Q} defined by $a * b = ab + 1$ associative? Justify.
6. Define a binary operation on a set. Give an example.
7. State and prove change of scale property of Laplace Transform
8. Define periodic functions. Check weather the function $\sin ax$ is periodic
9. Write the formula for Fourier series of a function with period $2l$ defined in the interval $[0, 2l]$
10. Write the fourmula for half range Fourier cosine series of a function defined in the interval $[0, l]$
11. Define the Fourier Sine Transform.
12. Is \mathbb{Z}^+ with binary operation $+$ a group? Justify your answer.

(2 x 10 = 20)**PART B****Answer any 5 (5 marks each)**

13. Express $f(x) = |x|$, $-\pi < x < \pi$ as a Fourier series
14. Find the Fourier sine transform of $2e^{-5x} + 5e^{-2x}$.
15. Obtain Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi} & 0 \leq x \leq \pi \end{cases}$
16. Solve $y'' - 3y' + 2y = 4t + e^{3t}$, when $y(0) = 1$ and $y'(0) = -1$
17. Find the inverse Laplace transform of $\frac{2s^2 - 1}{(s^2 + 1)(s^2 + 4)}$
18. Find the Fourier cosine transform of e^{-x^2} .
19. Explain the group S_3 . Write down its elements and its subgroups.
20. Define $*$ on \mathbb{Q}^+ by $a * b = ab/2$. Show that $\langle \mathbb{Q}^+, * \rangle$ is group.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Find the (complex) Fourier Transform of $f(x) = \begin{cases} 1 - x^2, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1 \end{cases}$ and use it to evaluate $\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$
22. Obtain the half range cosine and sine series for $f(x) = x$ in the interval $0 \leq x \leq \pi$
23. a) Solve $\frac{d^2x}{dt^2} + x = t \cos 2t$, $x(0) = x'(0) = 0$
b) Find the Laplace transform of $\frac{1 - \cos t}{t^2}$
24. (a) Let S be the set of all real numbers except -1 . Define $*$ on S by $a * b = a + b + ab$. Show that $\langle S, * \rangle$ is a group.
(b) Show that the left and right cancellation laws hold in any group $\langle G, * \rangle$.
- (10 x 3 = 30)**