

**B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024**  
**SEMESTER 2 - COMPLEMENTARY MATHEMATICS FOR PHYSICS AND CHEMISTRY**  
**COURSE : 19U2CPMAT2 - CALCULUS – 2 AND NUMERICAL ANALYSIS**  
*(For Regular - 2023 Admission and Improvement / Supplementary – 2022/2021/2020/2019 Admissions)*

Time : Three Hours

Max. Marks: 75

**PART A****Answer any 10 (2 marks each)**

1. Define Simpson's three eighth rule.
2. Show that  $\Delta[af(x) + bg(x)] = a\Delta f(x) + b\Delta g(x)$ .
3. If  $\vec{r} = xi + yj + zk$ , show that  $grad\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$
4. Show that  $\Delta c = 0$ , where  $c$  is constant.
5. Write the condition for convergence of Newton-Raphson method.
6. If  $\vec{r} = \sin ti + \cos t j + tk$ , find  $\left|\frac{d^2 \vec{r}}{dt^2}\right|$
7. State Green's theorem in the plane.
8. Find a unit vector normal to the surface  $x^3 + y^3 + 3xyz = 3$  at the point  $(1, 2, -1)$
9. Write the formula for Newton-Raphson method.
10. Given first two approximations  $a$  and  $b$ , what is the next approximation using bisection method?
11. Find a unit vector normal to the surface  $xy^3z^2 = 4$  at the point  $(-1, -1, 2)$
12. Show that  $\Delta[f(x).g(x)] = f(x+h)\Delta g(x) + g(x)\Delta f(x)$ .

**(2 x 10 = 20)****PART B****Answer any 5 (5 marks each)**

13. The following table gives the corresponding values of  $x$  and  $y$ . From the difference table express  $y$  as a function of  $x$

$x$	0	1	2	3	4
$y$	3	6	11	18	27

14. If  $\vec{r} = xi + yj + zk$  prove that  $div\left(r^n \vec{r}\right) = (n+3)r^n$
15. Evaluate  $\oint_C \vec{F} \cdot d\vec{r}$ , by Stoke's Theorem, where  $\vec{F} = y^2 \hat{i} + x^2 \hat{j} - (x+z)\hat{k}$  and  $C$  is the boundary of the triangle with vertices  $(0, 0, 0)$ ,  $(1, 0, 0)$  and  $(1, 1, 0)$ .
16. Define Shift operator and prove that  $xy_1 + x^2y_2 + x^3y_3 + \dots = \frac{x}{1-x}y_1 + \left(\frac{x}{1-x}\right)^2 \Delta y_2 + \left(\frac{x}{1-x}\right)^3 \Delta^2 y_2 + \dots$
17. Compute the real roots of  $x \log_{10} x - 1.2 = 0$ , correct to three decimal places.
18. Find the real root of the equation  $xe^x - 3 = 0$  by Regula-Falsi method, correct to three decimal places.

19. Prove that the vector  $f(r) \vec{r}$  is irrotational.
20. Find  $\iint_S \vec{F} \cdot \hat{n} dS$ , where  $\vec{F} = (2x + 3z)\hat{i} - (xz + y)\hat{j} + (y^2 + 2z)\hat{k}$  and  $S$  is the surface of the sphere having centre at  $(3, -1, 2)$  and radius 3.

**(5 x 5 = 25)**

**PART C**

**Answer any 3 (10 marks each)**

21. If  $\vec{r} = xi + yj + zk$ , prove that (i)  $\text{div} \left( \frac{\vec{r}}{r^3} \right) = 0$  (ii)  $\nabla^2 (r^n) = n(n + 1)r^{n-2}$
22. Evaluate  $\iint_S \vec{A} \cdot \hat{n} dS$ , where  $\vec{A} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$  and  $S$  is the surface of the plane  $2x + y + 2z = 6$  in the first octant.
23. Use Newton's divided difference formula to evaluate  $f(8)$  given:

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

24. Using bisection method, find the negative root of  $x^3 - x + 11 = 0$ , correct to four decimals.

**(10 x 3 = 30)**