Reg. No .... Name **24U232**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024

SEMESTER 2 - MATHEMATICS

COURSE : **19U2CRMAT2 - ADVANCED CALCULUS AND TRIGONOMETRY**

(For Regular - 2023 Admission and Improvement / Supplementary – 2022/2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Find parametric equations for the portion of the right circular cylinder $x^2 + z^2 = 9$ for which $0 \le y \le 5$
in terms of the parameters u and v.
2. Separate into real and imaginary parts the expression sin (α+iβ)
3. Find the polar coordinates of the point P whose rectangular coordinates are $\left(-2,\ -2\sqrt{3}\right)$.
4. Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ at the given point without eliminating the parameter where $x = \sqrt{t},\ y = 2t + 7\ ;\ t = 1$.
5. Find the n^{th} derivative of $\sin(ax + b)$.
6. Find the n^{th} derivative of $\cos(ax + b)$.
7. Define simple polar region with example
8. Write any two properties of double integrals.
9. Prove that sin ix = i sin hx
10. Find the asymptotes parallel to the coordinate axes of the curve $x^2y - 3x^2 - 5xy + 6y + 2 = 0$.
11. State the symmetry tests in polar coordinates.
12. Find the arc length of the spiral $r = e^{\theta}$ between $\theta = 0$ and $\theta = \pi$.

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. Separate $log(\alpha + i\beta)$ into real and imaginary parts.
14. Find the area of the region enclosed by the cardioid $r = 2 + 2\ \cos\theta$.
15. Evaluate $\int_0^1 \int_{-x}^{x^2} y^2x\ dydx$
16. Find the area of the region enclosed in the first quadrant within the cardioid $r = 1 + \sin\theta$.
17. If x is real , show that $\sin h^{-1}x\ =\ \log\left[x\ +\ \sqrt{x^2+1}\right]$
18. Expand $\left(\sin^{-1}x\right)^2$ in ascending powers of x.
19. Show that for the curve x = α cosθ (1+sinθ), y = α sinθ (1+cosθ), the radius of curvature is a, at the point for which the value of the parameter $\theta = -\frac{\pi}{4}$.
20. Find the area of the region R enclosed between the parabola $\frac{1}{2}x^2$ and the line y = 2x.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. (i) Use a polar double integral to find the area enclosed by the three-petaled rose r = sin 3θ.

    (ii) Use polar coordinates to evalute $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} (x^2+y^2)^{\frac{3}{2}}\ dydx.$

22. Sum to infinity $c\ \sin\ \alpha\ + \frac{1}{2}c^2 \sin\ 2\alpha + \frac{1}{3}c^3 \sin\ 3\alpha + \ldots,\ \ |c| < 1$

23. Prove Leibnitz theorem. If $y = \left(x^2 - 1\right)^n$, prove that $\left(x^2 - 1\right)y_{n+2} + 2xy_{n+1} - n\left(n+1\right)y_n = 0.$

24. (a)Find the surface area of that portion of the surface$z = \sqrt{4 - x^2}$ that lies above the rectangle R in the xy- plane whose coordinates satisfy 0 $\leq$ x $\leq$ 1 and 0 $\leq$ y $\leq$4.

    (b)Find an equation of the tangent plane to the parametric surface x = uv , y = u, z = v^2 at the point where u = 2 and v = - I .

(10 x 3 = 30)