Reg. No .... Name **24U258**

END SEMESTER EXAMINATION - MARCH 2024

SEMESTER 2 - INTEGRATED M.Sc. PROGRAMME COMPUTER SCIENCE

COURSE : **21UP2CPCMT02 - MATHEMATICS - II - LINEAR ALGEBRA**

(For Regular - 2023 Admission and Improvement / Supplementary - 2022/2021 Admissions)

Time : Three Hours Max. Weightage : 30

PART A

Answer any 8 Questions

1. (-1)v = -v for every v ∈ V
2. Define Basis of a vector space and give the standard basis for F^2,P^2 and F^3
3. Check the list (2,3,1) , (1,-1,2) , (7,3,8) are linearly independent or not in F^3;
4. Define Linear Function with an Example.
5. Find the transpose of the following matrices;

   $$\begin{pmatrix} 2 & 5 & 3 \\ 1 & 8 & 0 \end{pmatrix}, \begin{pmatrix} a & c \\ d & c \\ b & b \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 & 4 & 6 & 5 \\ 1 & 3 & 5 & 0 & 1 \end{pmatrix}$$

6. Explain diagonal of a matrix and upper triangular matrix with suitable example.
7. Define Invariant subspace.Suppose T ∈ L(V) .Show that each of the following subspaces of V is invarient under T;

   1. {0}
   2. V
   3. NullT
   4. RangeT

8. prove the following result;
   < u, λv > = $\bar{\lambda}$ < u,v > for all λ ∈F and u,v ∈ V
9. Define innerproduct space
10. Prove the following result;

    1. For each fixed u ∈V, the function that takes V to < v,u > is a linear map from V to F
    2. < 0,u> = 0 for every u ∈ V

(1 x 8 = 8 Weight)

PART B

Answer any 6 Questions

11. Prove that a subset U of V is a subspace of V if and only if U satisfies the following three conditions:

    1. 0 ∈ U
    2. u,w ∈ U implies u+w ∈ U
    3. a ∈ F and u ∈ U implies au ∈ U

12. Check whether the list of vectors (1,1),(1,-1) is basis in F^2

13. Suppose $D \in L(P(R),P(R))$ is the differentiation map defined by $D_p = p'$ and $T \in L(P(R),P(R))$ is the multiplication by x^2 defined by $T_p(x) = x^2p(x)$.Then prove that multiplication on linear map is not commutative.

14. show that the following maps are linear;

1. T: V→W defined by T(v)=0
2. D: P(R)→P(R) defined by $D_p=p'$
3. T: P(R)→P(R) defined by $T_p= \int_0^1 P(x)\,dx$
4. T: P(R)→P(R) defined by $T_p(x) = x^2\,p(x)$
5. T: $R^3 \to R^2$ defined by T(x,y,z) = (2x-y+3z, 7x+5y-6z)

15. Check whether $T \in L(P^2)$ defined by $T(at^2+bt+c) = (5a+b+2c)t^2 + 3bt + (2a+b+5c)$ is diagonalizable with respect to the basis $t^2-2t, -2t+1, t^2+1$ of P^2.Give valuable reason for your answer.

16. Give matrix representarion for the following operators

1. $T \in L(F^3)$ defined by T(x,y,z) = (2x+y, 5y+3z, 8z)
2. $T \in L(F^2)$ defined by T(x,y) = (2x+3y, 5x)

17. State and prove Pythegorean theorem.

18. Check the standard basis in F^3 is an orthonormal list.

(2 x 6 = 12 Weight)

PART C

Answer any 2 Questions

19. The span of list of vectors in V is the smallest subspace of V containing all the vectors in the list.

20. Prove the following;

1. Let $T \in L(V,W)$ .Then T is injective if and only if NullT ={0}
2. If $T \in L(V,W)$ then range T is a subspace of W.

21. Suppose $T \in L(V)$ and $v_1,.......,v_n$ is a basis of V. Then the following are equivalent:
a) The matrix T with respect to $v_1,.......,v_n$ is upper triangular
b) $Tv_j \in$ span $(v_1,.......,v_n)$ for each j=1,2,....n.
c) span $(v_1,.......,v_n)$ is invariant under T for each j=1,2,....n.

22. State and prove Triangle inequality and Parallelogram Inequality.

(5 x 2 = 10 Weight)