Reg. No .... Name **24P2031**

M. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024

SEMESTER 2 - MATHEMATICS

COURSE : **21P2MATT08 - GRAPH THEORY**

(For Regular 2023 Admission and Improvement/Supplementary 2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Prove or disprove: Let G be a simple connected graph with $n \geq 3$.Then G has a cut edge if and only if it has a cut vertex. (A, CO 2)
2. Prove that a simple graph G with n vertices $n \geq 2$, is complete if and only if $\kappa(G) = n - 1$ (U, CO 2)
3. Define a graphical sequence.Show that the sequence $(7, 6, 3, 3, 2, 1, 1, 1)$ is not graphical. (U, CO 1)
4. Define a self complementary graph and give an example. (U, CO 1)
5. Define proper vertex coloring and chromatic number of a graph G. (R, CO 3)
6. Let f be a plane graph and f be a face of G. Show that there exists a plane embedding of G in which f is the exterior face. (U, CO 4)
7. Give an example of a cubic graph with edge chromatic number 4. (A, CO 3)
8. If $(d_1, d_2, \ldots, d_n)$ is the degree sequence of a graph, and r be any positive integer.
   Show that $\sum_{i=1}^{n} d_i^r$ is even. (E, CO 1)
9. Show that the Petersen graph is nonplanar. (An, CO 4)
10. If u and v are nonadjacent vertices of a tree T, show that $T + uv$ contains a unique cycle. (U, CO 2)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Show that the closure of a graph is well defined. (An, CO 3)
12. Prove that a vertex v of a tree T with at least three vertices is a cut vertex of T if and only if v is not a pendant vertex. (A, CO 2)
13. Define the dual H of a plane graph G. Further,describe the method of drawing the canonical embedding G^* of G in the plane. (An, CO 4)
14. Show that every tournament of order n has at most one vertex with $d^+(v) = n - 1$. (A, CO 1)
15. Show that if G is a simple planar graph with at least three vertices, then $m \leq 3n - 6$ (A, CO 4)
16. Show that a simple connected graph contains at least $m - n + 1$ distinct cycles. (E, CO 1)
17. Show that for a simple bipartite graph, $m \leq \frac{n^2}{4}$. (A, CO 1)
18. Briefly describe Hamilton's "Around the World Game" and its significance. (R, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Show that a graph G is planar if and only if each of its blocks is planar. (A, CO 4)
20. Show that a graph G is Eulerian if and only if it has an odd number of cycle decompositions. (An, CO 3)
21. Find the number of spanning trees of the labeled graph K_4. (A, CO 2)
22. If the simple graphs G_1 and G_2 are isomorphic, show that $L(G_1)$ and $L(G_2)$ are isomorphic. Is the converse true? Justify. (E, CO 1)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain basic concepts such as subgraphs, degrees of vertices, paths and connectedness, automorphisms of a simple graph, line graphs and basic concepts of tournaments.	E	3, 4, 8, 14, 16, 17, 22	14
CO 2	Comprehend connectivity, vertex cuts , edge cuts, connectivity and edge connectivity, blocks, counting the number of spanning trees and Cayley's formula.	E	1, 2, 10, 12, 21	10
CO 3	Analyse vertex and edge independent sets, Eulerian graphs, Hamiltonian graphs, vertex colorings and related results.	E	5, 7, 11, 18, 20	11
CO 4	Explain edge coloring and planarity, certain definitions and properties, dual of a plane graph, the four color theorem and the Heawood five color theorem.	E	6, 9, 13, 15, 19	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;