Reg. No .... Name **24U426**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024

SEMESTER 4 - MATHEMATICS

COURSE : **19U4CRMAT4 - ANALYTIC GEOMETRY NUMERICAL METHODS AND NUMBER THEORY**

(For Regular - 2022 Admission and Improvement / Supplementary - 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Find the polar equation of the circle which passes through the pole and two points whose polar coordinates are $(d, 0)$ and $\left(2d, \frac{\pi}{3}\right)$. Find also the radius of the circle.
2. Give an example of algebraic equation.
3. Find the equation of the directrix of the conic $r \sin^2 \frac{\theta}{2} = 1$.
4. Find the polar equation of the line passing through (r_1, θ_1) and (r_2, θ_2).
5. Use the NewtonRaphson method to obtain a root, correct to two decimal places, of the equation $\cot x = -x$.
6. Find the angle between the lines $x^2 + xy - 6y^2 = 0$.
7. Show that if $a \equiv b(\mod n)$ and $c \equiv d(\mod n)$, then $a + c \equiv b + d(\mod n)$.
8. Find the nature of the conic $\frac{5}{r} = 2 - 2\cos\theta$.
9. Define the rank of a second order curve.
10. Show that if p is a prime and $k > 0$, then $\phi(p^k) = p^k - p^{k-1}$.
11. State the converse of Fermat's theorem.
12. Find the equation of the ellipse which has the point $(-1, 1)$ for a focus, the line $4x - 3y = 0$ the corresponding directrix and whose eccentricity is $\frac{5}{6}$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Use the method of false position to find a real root, correct to three decimal places, of the equation $x^3 - x - 4 = 0$.
14. The equation $2x = \log_{10} x + 7$ has a root between 3 and 4. Find this root, correct to three decimal places, by regula-falsi method.
15. For given integers a, b, c, prove that gcd$(a, bc) = 1$ if and only if gcd$(a, b) = 1$ and gcd$(a, c) = 1$.
16. Find the equation of the hyperbola whose asymptotes are $x + 2y + 3 = 0$ and $3x + 4y + 5 = 0$ and which passes through the point $(1, -1)$.
17. Find the asymptotes of $2x^2 + 5xy + 2y^2 + 4x + 5y = 0$.
18. Show that the locus of a point which moves such that the difference of its distances from two fixed points is a constant , is a hyperbola.
19. Find the centre, foci and lengths of axes of the hyperbola $x^2 - 2y^2 - 2x + 8y - 1 = 0$.
20. Let $n = p_1 p_2 \ldots p_r$ be a composite square-free integer, where the p_i are distinct primes. If $p_i - 1 | n - 1$ for $i = 1, 2, \ldots, r$, then prove that n is an absolute pseudoprime.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Using NewtonRaphson method, derive a formula for finding the k^{th} root of a positive number N and hence compute the value of $25^{1/4}$.

22. Derive a formula to evaluate $\phi(n)$. Hence prove that if the integer n has r distinct odd prime factors, then $2^r|\phi(n)$.

23. Show that for the conic $\frac{l}{r} = 1 + e\cos\theta$, the equation to the directrix corresponding to the focus other than the pole is $\frac{l}{r} = -\frac{1-e^2}{1+e^2}e\cos\theta$.

24. Show that the equation $7x^2 - 48xy - 7y^2 - 20x + 140y + 300 = 0$ represents a hyperbola and find its canonical equation.

(10 x 3 = 30)