Reg. No .... Name **24P2018**

M. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024

SEMESTER 2 - MATHEMATICS

COURSE : **21P2MATT07 - COMPLEX ANALYSIS**

(For Regular 2023 Admission and Improvement/Supplementary 2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	Evaluate $\int_\gamma \frac{e^z - e^{-z}}{z^n} dz$ where n is a positive integer and $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi$	(A, CO 2)
2.	Define cross ratio? Evaluate the cross ratio $(7+i, 1, 0, \infty)$	(A, CO 1)
3.	Show that a Mobius transformation takes circles onto circles.	(R, CO 1)
4.	Evaluate $\int_\gamma \frac{dz}{z-a}, \gamma(t) = a + re^{it}, 0 \leq t \leq 2\pi$	(A, CO 2)
5.	For the $f(z) = exp(z^{-1})$ has an isolated singularity at $z = 0$. Determine its nature; if it is a removable singularity define $f(0)$ so that f is analytic at$z = 0$; if it is a pole find the singular part.	(An, CO 3)
6.	State Hadamard's three circles theorem.	(R, CO 4)
7.	Evaluate $\int_\gamma \frac{dz}{z^2+1}$ where $\gamma(t) = 2e^{it}, 0 \leq t \leq 2\pi$	(A, CO 2)
8.	Explain different types of singularities with examples.	(U, CO 3)
9.	Evaluate 1) $(0, 1, i, -1)$ 2) $(i-1, \infty, 1+i, 0)$	(An, CO 1)
10.	State the second version of The Maximum Modulus theorem and give the importance of boundedness in it.	(A, CO 4)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	Define symmetric points. explain symmetric points w.r.t a straight line.	(A, CO 1)
12.	Let G be either the whole plane $\mathbb{C}$ or some open disk. If $u : G \to \mathbb{R}$ is a harmonic function then prove that u has a harmonic conjugate.	(A, CO 1)
13.	State and prove the Schwarz's lemma.	(R, CO 4)
14.	State and prove the second version of Cauchy's integral formula.	(R, CO 2)
15.	State and prove argument principle.	(U, CO 3)
16.	Let $f(z) = \frac{1}{z^2(4z-1)}$ find the Laurent series expansion valid in the region a) $0 \leq \lvert z\rvert \leq 1/4$ and b) $\lvert z\rvert \geq 1/4$	(A, CO 3)
17.	State and prove the first version of The Maximum Modulus theorem.	(U, CO 4)
18.	Let G be an open subset of the plane and $f : G \to \mathbb{C}$ an analytic function. If $\gamma_1, \gamma_2 . . . . \gamma_m$ are closed rectifiable curves in G such that $n(\gamma_1; w) + n(\gamma_2; w) + ... n(\gamma_m; w) = 0$ for all w in $\mathbb{C} - G$ then show that for a in $G - \gamma$ and $k \geq 1$, $f^k(a) \sum_{j=1}^{m} n(\gamma_j, a) = k! \sum_{j=1}^{m} \frac{1}{2\pi i} \int_{\gamma_j} \frac{f(z)}{(z-a)^{k+1}} dz$	(A, CO 2)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ have radius of convergence $R > 0$.Then:
(a) For each$k \geq 1$ the series
$\sum_{n=k}^{\infty} n(n-1)\ldots\ldots(n-k+1)a_n(z-a)^{n-k}$ has radius of convergence R;
(b) The function $f$is infinitely differentiable on $B(a;R)$ and, furthermore, $f^k(z)$ is given by the series
$\sum_{k}^{\infty} n(n-1)\ldots\ldots(n-k+1)a_n(z-a)^{n-k}$ for all $k \geq 1$ and $|z-a| \leq R$
(c) For $n \geq 0$, $a_n = \frac{1}{n!} f^n(a)$, (A, CO 1)

20. Let γ be a rectifiable curve and suppose ϕ is a function defined and continuous on γ. For each $m \geq 1$ let $F_m(z) = \int_\gamma \phi(w)(w-z)^{-m} dw$ for $z \notin \gamma$. Then show that for each F_m is analytic on $\mathbb{C} - \gamma$ and $F'_m(z) = mF_{m+1}(z)$ (U, CO 2)

21. Let $f : D \longrightarrow D$ be one-one analytic map of$D$ onto itself and suppose $f(a) = 0$. Then prove that there is a complex number c with $|c| = 1$ such that $f = c\phi_\alpha$ (A, CO 4)

22. Derive the Laurent series development. (R, CO 3)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Analyze Analytic functions and Mobius transformations	U	2, 3, 9, 11, 12, 19	12
CO 2	Explain power series representation of analytic functions, Cauchy's integral formula, Cauchy's theorem	U	1, 4, 7, 14, 18, 20	12
CO 3	Illustrate about different types of singularities, residues and Rouche's theorem	U	5, 8, 15, 16, 22	11
CO 4	Explain Maximum Modulus theorem, maximum principle, Schwarz's lemma, convex functions and Hadmard's Three Circles Theorem.	U	6, 10, 13, 17, 21	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;