Reg. No .... Name **24P4045**

M. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024

SEMESTER 4 - MATHEMATICS

COURSE : **21P4MATTEL20 -THEORY OF WAVELETS**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. For $Z \in l^2(Z_n)$, define $T(Z) \in (Z_N)$ by $(T(z))(n) = z(n-1)$ for all 'n'. Prove that T is translation invariant. (R, CO 1)
2. If $z, w \in l'(z)$, prove that $\sum\limits_{n\in z} z(m-n)w(n)$ converges absolutely for each $m \in z$. (A, CO 3)
3. Show that if $\sum\limits_{n\in Z} w(n)$ converges absolutely, then $\lim\limits_{n\to\infty} w(n) = 0$ and $\lim\limits_{n\to\infty} w(-n) = 0$. (An, CO 3)
4. Determine the total size of the output vector of the analysis phase of the Pth stage filter bank. (An, CO 2)
5. Define Convolution operator. (R, CO 1)
6. Suppose $z \in l^2(Z)$. Then define $\tilde{z}(n)$ and $z^*(n)$ for all $n \in Z$. (U, CO 4)
7. If $\psi_{-j,k} = R_{2^jk} f_i$ and $\phi_{-j,k} = R_{2^jk} g_j$, prove that $\hat{\phi}_{-j,k}(m) = e^{-2\pi i 2^j km/N} \hat{\psi}_{-j,0}(m)$ and $\hat{\phi}_{-j,k}(m) = e^{-2\pi i 2^j km/N}\, \hat{\phi}_{-j,0}(m)$. (E, CO 2)
8. Suppose $z, w \in l^2(Z)$. Prove that $D(z) * w = D(z * U(w))$. (A, CO 4)
9. If $z = (1, 2, 3, i) \in l^2(Z_4)$, find $(R_2 z)(3)$ and $(R_2 z)(100)$. (An, CO 1)
10. Suppose $M \in Z$and $z \in C$. When we say a sequence $\{z_n\}_{n=M}^{\infty}$ of complex numbers converges to z? When we say this sequence is a Cauchy Sequence? (E, CO 3)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Suppose $\theta_0 \in (-\pi, \pi)$ and $\alpha > 0$ is sufficiently small that $-\pi < \theta_0 - \alpha < \theta_0 + \alpha < \pi$ .Define $I = (\theta_0 - \alpha, \theta_0 + \alpha)$ and J=$(\theta_0 - \frac{\alpha}{2}, \theta_0 + \frac{\alpha}{2})$. Then prove that there exists a $\delta > 0$ and a sequence of real valued trigonometric polynomials $\{p_n(\theta)\}_{n=1}^{\infty}$ such that (U)
    (i) $p_n(\theta) \geq 1$ for $\theta \in I$.
    (ii) $p_n(\theta) \geq (1+\delta)^n$ for $\theta \in J$.
    (iii) $|p_n(\theta)| \leq 1$ for $\theta \in [-\pi, \pi) - I$.
12. Suppose N is divisible by 2^p . Suppose $u_1, v_1, \in l^2(Z_N)$ are such that the system matrix A(n) of u_1 and v_1 is unitary for all n.
    Define $u_l(n) = \Sigma_{k=0}^{2^{l-1}-1} u_1(n + \frac{kN}{2^{l-1}})$ and $v_l(n) = \Sigma_{k=0}^{2^{l-1}-1} v_1(n + \frac{kN}{2^{l-1}})$ for $l = 2, 3, \ldots, p$. If we define $f_1 = v_1$ and $g_1 = u_1$ and for $l = 2, 3, .., p$ if we define $f_l = g_{l-1} * U^{l-1}(v_l)$ and $g_l = g_{l-1} * U^{l-1}(u_l)$,
    Then prove that U_1, V_1, U_2, V_2.... U_p, V_p is a Pth stage wavelet filter sequence. (An, CO 2)
13. (a) If N=2M, when M is a positive integer, then define z^* for any $z \in l^2(Z_N)$
    (b) Prove that $(z^*)^{\wedge}(n) = \hat{z}(n+M)$ for all n. (A, CO 1)

14. Suppose $z \in l^2(Z)$ and $w \in l^1(Z)$. Then prove that $z * w \in l^2(Z)$ and $||z * w|| \leq ||w||, ||z||$. (An, CO 4)

15. Prove that $l^2(z)$ is a vector space. (U, CO 3)

16. Let $b \in l^2(Z_N)$ and $T_b : l^2(Z_N) \to l^2(Z_N)$ be defined by $T_b(z) = b * z$. Then prove that T_b is translation invariant linear transformation. (A, CO 1)

17. Suppose N is divisible by 2^l.
Define $u_l \in l^2(Z_{N/2}^{l-1})$ by $u_l(n) = \sum_{k=0}^{2^{l-1}-1} u_1(n + \frac{kN}{2^{l-1}})$. Then prove that $\hat{u_l}(m) = \hat{u_1}(2^{l-1}m)$. (An, CO 2)

18. Describe the first stage real shannon basis for $l^2(Z_N)$ if N is divisible by 4. (U, CO 1)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Suppose l is a positive integer, $g_{l-1} \in l^2(Z)$ and $\{R_{2^{l-1}k}g_{l-1}\}_{k\in Z}$ is orthonormal in $l^2(Z)$. Suppose that $u, v \in l^1(Z)$ and the system matrix A(θ) of u and v is unitary for all θ . Define $f_l = g_{l-1} * U^{l-1}(v)$ and $g_l = g_{l-1} * U^{l-1}(u)$. Then prove that $\{R_{2^lk}f_l\}_{k\in Z} \bigcup \{R_{2^lk}g_l\}_{k\in Z}$ is orthonormal. (U, CO 4)

20. (a) Describe first stage shannon basis for $l^2(Z_N)$ if N is divisible by 4.
(b) Deduce the first stage real shannon basis for $l^2(Z_N)$. (U, CO 1)

21. Suppose $T : L^2([-\pi, \pi)) \to L^2([-\pi, \pi))$ is a bounded translation invariant linear transformation . Then prove that $T(e^{im\theta}) = \lambda_m e^{im\theta}$ for some $\lambda_m \in C$ and it is true for all $m \in Z$. (An, CO 3)

22. Suppose N is divisible by 2^l, $g_{l-1} \in l^2(Z_N)$ and the set $\{R_{2^{l-1}k}g_{l-1}\}_{k=0}^{\frac{N}{2^{l-1}-1}}$ is orthonormal and has $\frac{N}{2^{l-1}}$ elements. Suppose $u_l, v_l \in l^2(Z_{N/2}^{l-1})$ and the system matrix $A_l(n)$ is unitary for all $n = 0, 1, 2, \ldots, (N/2^l) - 1$. Define $f_l = g_{l-1} * U^{l-1}(v_l)$ and $g_l = g_{l-1}(*)U^{l-1}(u_l)$. With the usual notations prove that $V_{-l} \oplus W_{-l} = V_{-l+1}$ (An, CO 2)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Define first stage and pth stage wavelet basis for l 2[ZN], Fourier transform including discrete case ,complete orthonormal system , first stage wavelet system and homogeneous wavelet system for l2[Z]	U	1, 5, 9, 13, 16, 18, 20	14
CO 2	Explain the filter bank diagram and its use in the construction of the output of the analysis phase of the filter bank	U	4, 7, 12, 17, 22	11
CO 3	Apply theory of wavelets in the frequency analysis of a video or audio signal.	U	2, 3, 10, 15, 21	10
CO 4	Develop wavelet bases for l 2 [ZN] and l2 [Z],both first stage and pth stage	U	6, 8, 14, 19	9

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;