Reg. No .................... Name .............................. **24P2003**

**M. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024**

**SEMESTER 2 - MATHEMATICS**

COURSE : **21P2MATT06 - BASIC TOPOLOGY**

*(For Regular 2023 Admission and Improvement/Supplementary 2022/2021 Admissions)*

Duration : Three Hours Max. Weights: 30

**PART A**

**Answer any 8 questions** **Weight: 1**

1. Let $X$ be a connected space. Show that only clopen subsets of $X$ are $X$ and $\phi$. (A, CO 3)
2. Does unioin of any two topologies form a topology? Justify. (R, CO 1)
3. Let $f: X \to Y$ be a continuous surjection, then if $X$ is connected so is $Y$. (U, CO 3)
4. Prove that a constant function is always continuous. (U, CO 2)
5. Distinguish between absolute property and relative property. Give an example to each. (U, CO 2)
6. Let $X$ be an infinite set where the closed sets are finite sets. Is $X$ Hausdorff ? (U, CO 4)
7. Give an example of a topological space which is $T_1$ but not $T_2$. (R, CO 4)
8. Define $i^{th}$ projection and its subbase. (R, CO 2)
9. Define a metrisable space. Give an example of a space which is not metrisable. (U, CO 1)
10. Prove that in a discrete space if a sequence is convergent then it will be an eventually constant sequence. (R, CO 1)

**(1 x 8 = 8)**

**PART B**

**Answer any 6 questions** **Weights: 2**

11. Prove that every compact Hausdorff space is $T_3$. (R, CO 4)
12. Let $\tau_1$ and $\tau_2$ be topologies on a set $X$ and $Y \subset X$. If $\tau_1$ is stronger than $\tau_2$, prove that $\tau_1/Y$ is stronger than $\tau_2/Y$. (R, CO 1)
13. Prove that if a subset of $\mathscr{R}$ is not an interval then it is not connected. (U, CO 3)
14. Prove that product topology is the weak topology determined by the projection functions. (U, CO 2)
15. Prove that the topological product of two connected space is connected. (U, CO 3)
16. Show that every open surjective map is a quotient map. (R, CO 2)
17. Define metric topology. Prove that every metric space is a topological space, where the topology is metric topology. (R, CO 1)
18. Prove that every compact Hausdorff space is $T_4$. (U, CO 4)

**(2 x 6 = 12)**

**PART C**

**Answer any 2 questions** **Weights: 5**

19. Let $[(X_i, \mathscr{T}_i), i = 1, 2, \ldots n]$ be a collection of topological spaces and $(X, \mathscr{T})$ their topological product. Prove that each projection $\pi_i$ is continuous. Also show that if $Z$ is any space then the fuction $f: Z \to X$ is continuous if and only if $\pi_i o f: Z \to X_i$ is continuous for all $i = 1, 2, \ldots n$. (U, CO 2)

20. Define $T_4$ and prove that all metric spaces are $T_4$. (U, CO 4)

21. Define boundary, closure and interior of a set. Prove that $\mathscr{R}$ under usual metric forms a topological space. (A, CO 1)

22. State and prove equivalent condition for locally connected space. (U, CO 3)

**(5 x 2 = 10)**

OBE: Questions to Course Outcome Mapping

| CO | Course Outcome Description | CL | Questions | Total Wt. |
|---|---|---|---|---|
| CO 1 | Classify and compare different topological spaces | U | 2, 9, 10, 12, 17, 21 | 12 |
| CO 2 | Continuous functions and Quotient map | U | 4, 5, 8, 14, 16, 19 | 12 |
| CO 3 | Connectedness | U | 1, 3, 13, 15, 22 | 11 |
| CO 4 | Separation Axiom | U | 6, 7, 11, 18, 20 | 11 |

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;