Reg. No .... Name **24U656**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024

SEMESTER 6 - MATHEMATICS

COURSE : **19U6CRMAT12- FOURIER SERIES, LAPLACE TRANSFORMS AND METRIC SPACES**

(For Regular - 2021 Admission and Supplementary - 2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Write the formula for half range Fourier cosine series of a function defined in the interval $[0,l]$
2. Write the formula for Fourier serier of a function with period $2l$defined in the interval $[0, 2l]$
3. Define periodic functions. Check whether the function $sinax$ is periodic.
4. Define Laplace transform and find the Laplace transform of t^n, n is an integer.
5. Find the Laplace transform of $e^{at} \cos at$
6. Find the Laplace transform of $t^2 sin2t$.
7. Give an example of a metric on $\mathbb{R}$ other than the usual metric.
8. Any metric space has two open subsets. True or False. Justify.
9. Define a metric space.
10. Give an example of a function which is continuous but not uniformly continuous on (a) $\mathbb{R}$ (b) $(0, 1)$.
11. Define a Cauchy sequence. Give example.
12. Limit of a sequence is also a limit point of the set of points of the sequence. True or false. Justify.

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. Find the Fourier series of $f(x) = 4 - x^2, -2 \leq x \leq 2$ and hence show that$\frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots.$
14. Expand $f(x) = e^{-x}$ as a Fourier series in the interval $(-l, l)$
15. Find the inverse Laplace transform of $\frac{s^2+1}{s^3+3s^2+2s}$.
16. Find the inverse Laplace transform of $\frac{21s-33}{(s+1)(s-2)^3}$.
17. Let X be a metric space and A be a subset of X. Prove that Int(A) is an open subset of A which contains every open subset of A.
18. Let X be a metric space and let A be a subset of X. Prove that
    1) A is closed $\Leftrightarrow A = \overline{A}$
    2) $\overline{A}$ equals the intersection of all closed supersets of A.
19. Let X be a complete metric space and Y be a closed subspace of X, then prove that Y is complete.
20. Let X and Y be metric spaces and fa mapping of X into Y. Prove that if f is continuous at x_0, then $x_n \rightarrow x_0 \Rightarrow f(x_n) \rightarrow f(x_0)$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Find the half range Fourier sine and cosine series of the function $f\left(x\right)=\begin{cases} x & 0<x<\frac{\pi}{2} \\ \pi-\text{x} & \frac{\pi}{2}<x<\pi \end{cases}$

22. a) Solve $y'' + y' - 2y = t, y\left(0\right) = 1,\ y'\left(0\right) = 0$

    b) Evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$

23. Let X be a metric space. Prove that

    1) Each open sphere is an open set

    2) A subset G of X is open $\Leftrightarrow$ it is a union of open spheres.

24. (a) When do we say that a set is nowhere dense?

    (b) State and prove the Baire's theorem.

(10 x 3 = 30)