Reg. No .... Name **24U638**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024

SEMESTER 6 - MATHEMATICS

COURSE : **19U6CRMAT11 - LINEAR ALGEBRA AND GRAPH THEORY**

(For Regular 2021 Admission and Supplementary 2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Define linearly dependent and linearly independent set of vectors in a vectorspace.
2. Determine whether the transformation **T: V → W** defined by T(v) = 0 for all vectors v in **V** is linear.
3. State and prove first theorem on graph theory.
4. Explain the travelling salesman's problem.
5. Draw the graph whose adjacency matrix is
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$
6. Determine whether u =[1 2 3] is a linear combination of v_1 = [1 1 1], v_2 = [2 4 0], and v_3 = [0 0 1]
7. Determine whether the transformation **T: V → V** defined by T(v) = kv for all vectors v in **V** and any scalar k, is linear.
8. Expain the Chinese postman problem.
9. Prove that for any vector space V, the subset containing only the zero vector is a subspace.
10. Let **T: R^2 → R^2** is defined by T [a b] = [a+2 b-2] . Find
a) T [2 3] b) T [-1 5]
c) T [-8 200] d) T [0 -7]
11. Define underlying simple graph with any example.
12. Define a Hamiltonian cycle and a Hamiltonian graph.

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

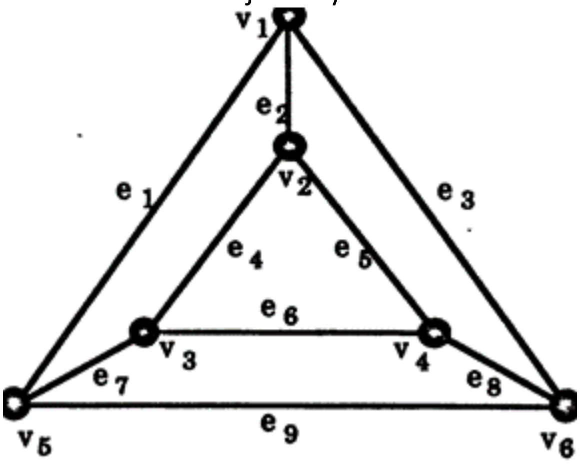
13. Determine whether the transformation **L** is linear if **L: P_3 → P_2** is defined by L($a_3t^3 + a_2t^2 + a_1t + a_0$) = $3a_3t^2 + 2a_2t + a_1$ where a_i (i = 0,1,2,3) denotes real number.
14. Let G be a simple graph with n vertices and let $\overline{G}$ be its complement.
(a) Prove that, for each vertex v in G, $d_G(v) + d_{\overline{G}}(v) = n — 1$.
(b) Suppose that G has exactly one even vertex. How many odd vertices does $\overline{G}$ have?
15. Find the matrix representation with respect to the standard basis in **R^2** and the basis D = { t^2+1, t+1, t-1 } in P^2 for the linear transformation **T: R^2 → P_2** defined by
$$T\begin{bmatrix} a \\ b \end{bmatrix} = (4a+b)t^2 + (3a)t + (2a-b)$$
16. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian

17. Determine whether the set of two-dimensional column matrices with all components real and equal is a vector space under regular addition but with scalar multiplication defined as

$$\alpha \odot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -\alpha a \\ -\alpha b \end{bmatrix}$$

18. a)Prove that it is imppossible to have group of nine people such that each one knows exactly five others in the group
b) Draw a graph which is not Euler but having an Euler trail

19. Write down the adjacency matrix and the incidence matrix for the graph



(b)

20. Determine whether the set

$$\mathbb{D} = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right\}$$

is a basis for $\mathbf{M}_{2\times 2}$

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Let G be a graph with n vertices.Then prove that the following statements are equivalent
(a) G is a tree
(b) G is acyclic graph with $n-1$ edges
(c) G is a connected graph with $n-1$ edges

22. State and prove Dirac theorem

23. Prove that for any linear transformation T from an n-dimensional vector space V to W, sum of rank of T and nullity of T is n, the dimension of the domain.

24. Find a basis for the span of the vectors in

$$\mathbb{C} = \{t^3 + 3t^2,\ 2t^3 + 2t - 2,\ t^3 - 6t^2 + 3t - 3,\ 3t^2 - t + 1\}$$

(10 x 3 = 30)