Reg. No .... Name **24P4027**

M. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024

SEMESTER 4 - MATHEMATICS

COURSE : **21P4MATTEL18 - PROBABILITY THEORY**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Define a random variable. (U, CO 2)
2. Define almost sure convergence. (R, CO 4)
3. A card is chosen at random from a deck of 52 cards. Let A be the event that the card is an ace and B , the event that it is a club. Are A and B independent? (U, CO 1)
4. A die is rolled twice. Let all the elementary events in $\Omega = (i,j) : i,j = 1,2,...,6$ be assigned the same probability. Let A be the event that the first throw shows a number ≤ 2, and B be the event that the second throw shows atleast 5. Find $P(A \cup B)$. (A, CO 1)
5. If $F(x) = \begin{cases} 0, x < 0 \\ x, 0 \leq x < \frac{1}{2} \\ 1, x \geq \frac{1}{2} \end{cases}$ find $P(X > \frac{1}{4}), P(\frac{1}{3} < X \leq \frac{3}{8})$. (A, CO 2)
6. State Cauchy Schwartz inequality and Minkowski's inequality. (U, CO 3)
7. Let Ω ={ HH, HT, TH, TT} and S be the class of all subsets of Ω. Define X by $X(HH) = 2, X(TH) = 1 = X(HT), X(TT) = 0$.Show that X is a random variable. (A, CO 2)
8. Define mutually or completely independent rv's. (R, CO 3)
9. Define truncated distribution. (R, CO 3)
10. Let $X_1, X_2, ..., X_n$ be sequence of random variables with corresponding DF's given by $F_n(x) = \begin{cases} 0, x < -n \\ \frac{x+n}{2n}, -n \leq x < n \\ 1, x \geq n \end{cases}$. Does F_n converge to a DF? (A, CO 4)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. State and prove the multiplication rule. (A, CO 1)
12. Examine whether WLLN holds for the sequence {X_k} of independent rv's defined as follows:
$P(X_k = \pm 2^k) = 2^{-(2k+1)}, P(X_k = 0) = 1 - 2^{-2k}$. (A, CO 4)
13. Compute $E(X), Var(X), E(X^n), n \geq 0$,an integer whenever they exist.
$f(x) = 1, \frac{-1}{2} \leq x \leq \frac{1}{2}, = 0, otherwise$ (A, CO 2)
14. If A, B, C are mutually independent events then prove that $A \cup B$ and C are also independent. (A, CO 1)

15. Consider the joint pdf $f(x,y) = xe^{-x(1+y)}, x \geq 0, y \geq 0$ and $= 0, otherwise$ of (X, Y). Then find $E(Y|x)$. (A, CO 3)

16. Examine whether WLLN holds for the sequence $\{X_k\}$ of independent rv's defined as follows: $P(X_k = \pm k) = \frac{1}{2\sqrt{k}}, P(X_k = 0) = 1 - \frac{1}{\sqrt{k}}$. (A, CO 4)

17. Define MGF. Let X have pdf $f(x) = \begin{cases} \frac{1}{2}e^{\frac{-x}{2}}, x > 0 \\ 0, otherwise \end{cases}$.The find the MGF of X. (An, CO 2)

18. A fair coin is tossed 3 times. Let X denote the number of heads in 3 tossings and Y be the difference in absolute value between the number of heads and number of tails. Find the jpmf and marginal pmf's. (A, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. State Bayes theorem. Five percent of patients suffering from a certain disease are selected to undergo a new treatment that is believed to increase the recovery rate from 30 percent to 50 percent. A person is randomly selected from these patients after the completion of the treatment and is found to have recovered. What is the probability that the patient received the new treatment? (E, CO 1)

20. a)Define convergence in probability and convergence in law.
b)Let c be a constant. Show that $X_n \xrightarrow{L} c \Longrightarrow X_n \xrightarrow{P} c$. (A, CO 4)

21. Show that the function defined by $f(x,y,z,u) = \frac{24}{(1+x+y+z+u)^5}, x > 0, y > 0, z > 0, u > 0$, and $= 0, otherwise$ is a joint pdf . Also find $P(X > Y > Z > U)$. (A, CO 3)

22. Let X be the rv defined on a probability space (Ω, S, P). Define F on R by $F(x) = Q(-\infty, x] = P(\omega : X(\omega \leq x)$ for all $x \in R$.Show that F is a distribution function of rv X. (An, CO 2)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	define the principal concepts about probability and evaluating it.	A	3, 4, 11, 14, 19	11
CO 2	explain the concept of a random variable and the probability distributions	A	1, 5, 7, 13, 17, 22	12
CO 3	Analyze the concept of function of rv's and multiple rv's	An	6, 8, 9, 15, 18, 21	12
CO 4	Analyze the concept of convergence of sequence of rv's	An	2, 10, 12, 16, 20	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;