Reg. No .... Name **24U622**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2024

SEMESTER 6 - MATHEMATICS

COURSE : **19U6CRMAT10 - COMPLEX ANALYSIS**

(For Regular 2021 Admission and Supplementary 2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Show that the existence of the derivative of a function at a point implies the continuity fo the function at that point.
2. Compute the Maclaurin series expansion of $f(z) = \sin z$.
3. Write the coefficient of z^{2n} in the Taylor series expansion of $\sin z$.
4. Define order of a pole.
5. Show that an antiderivative of a given function $f(z)$ is unique except for an additive constant.
6. Find the principal value of i^i.
7. Define isolated singularity.
8. True or false: $\int_C e^{z^3} dz = 0$ for any closed contour C. Justify.
9. State Jordan cuve theorem.
10. Write the Laurent series expansion of $e^{1/z}$ about $z = 0$.
11. Evaluate P.V. $\int_{-\infty}^{\infty} x dx$
12. Use definition to evaluate $f'(0)$, where $f(z) = |z|^2$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. State and prove Cauchy's inequality.
14. Prove that $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$ whenever $|z| < 1$.
15. Suppose that $\lim_{z \to z_0} f(z) = w_0$ and $\lim_{z \to z_0} F(z) = W_0$. Show that $\lim_{z \to z_0} [f(z)F(z)] = w_0 W_0$.
16. Compute the residue of $f(z) = \dfrac{1}{z(e^z - 1)}$ at $z = 0$.
17. Let C denote a contour of length L, and suppose that a function $f(z)$ is piecewise continuous on C. If M is a nonnegative constant such that $|f(z)| \leq M$ for all points z on C at which $f(z)$ is defined , then prove that $\left| \int_C f(z) dz \right| \leq ML$

18. Suppose that $z_n = x_n + iy_n$ and $S = X + iY$. Prove that $\sum_{n=1}^{\infty} z_n = S$ if and only if $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$.

19. Find the residues of the singularities of $f(z) = \dfrac{z^3 + 2z}{(z-1)^3}$.

20. Use definition to show that $\lim_{z \to 1} f(z) = \frac{i}{2}$, where $f(z) = \frac{i\bar{z}}{2}$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. State and prove Morera's theorem.

22. Show that the absolute convergence of a series of complex numbers implies the convergence of that series. Is the converse true. Justify

23. Prove that an isolated singular point z_0 of a function f is a pole of order m if and only if $f(z)$ can be written in the form $f(z) = \dfrac{\phi(z)}{(z-z_0)^m}$ where $\phi(z)$ is analytic and nonzero at z_0 and $\underset{z=z_0}{Res} f(z) = \frac{\phi^{(m-1)}(z_0)}{(m-1)!}$

24. Let $f(z) = u(x,y) + iv(x,y) \quad (z = x+iy)$ and $z_0 = x_0 + iy_0,\ w_0 = u_0 + iv_0$. Show that $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y) \to (x_0,y_0)} u(x,y) = u_0$ and $\lim_{(x,y) \to (x_0,y_0)} v(x,y) = v_0$.

(10 x 3 = 30)