Reg. No .... Name **23P144**

M. Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 1 : MATHEMATICS

COURSE : **21P1MATT04 : ORDINARY DIFFERENTIAL EQUATIONS**

(For Regular -2023 Admission and Improvement/Supplementary-2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Find the normal form of Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$, and use it to show that every nontrivial solution has infinitely many positive zeros. (A, CO 1)
2. Find the critical points of the system $\frac{dx}{dt} = y^2 - 5x + 6$, $\frac{dy}{dt} = x - y$. (U, CO 3)
3. Show that $\frac{d}{dx}(xJ_1(x)) = xJ_0(x)$. (A, CO 2)
4. Examine the existence and uniqueness of solution for the initial value problem $y' = x - y + 1, y(1) = 2$. (A, CO 4)
5. Express the function in terms of Legendre's polynomial $f(x) = x^4$. (E)
6. Define orthogonality of functions. (R, CO 1)
7. Evaluate $\int_{-1}^{1} xP_n^2(x)dx$. (U, CO 2)
8. Describe the phase portrait of the system $\frac{dx}{dt} = 1$, $\frac{dy}{dt} = 2$. (A, CO 3)
9. Calculate $(\frac{3}{2})!$ (U, CO 2)
10. Determine the nature and stability properties of the critical point (0, 0) for the linear autonomous system $\frac{dx}{dt} = -4x - y$, $\frac{dy}{dt} = x - 2y$. (A, CO 3)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Find the eigenvalues and eigenfunctions for the equation $y'' + \lambda y = 0, y(0) = 0, y(L) = 0$. (A, CO 1)
12. Express $J_4(x)$ in terms of J_0 and J_1. (A, CO 2)
13. Determine the nature and stability properties of the critical point (0, 0) for the linear autonomous system $\frac{dx}{dt} = 5x + 2y$, $\frac{dy}{dt} = -17x - 5y$. (A, CO 3)
14. Show that the Picard's theorem ensures a unique solution in the interval $|x| \leq 1/2$ for the initial value problem $dy/dx = x + y^2, y(0) = 0$. (A, CO 4)
15. Solve the vibrating string problem with initial shape $f(x) = \frac{x}{\pi}(\pi - x)$. (A, CO 1)
16. Show that $\int_{-1}^{1} P_m(x)P_n(x)dx = \frac{2}{2n+1}, m = n$. (A, CO 2)
17. Solve the following initial value problem upto third approximation. $dy/dx = 2x - y^2, y(0) = 0$ (A, CO 4)
18. Find the general solution of the system $\frac{dx}{dt} = x$, $\frac{dy}{dt} = y$. (An, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. State and prove orthogonality property of Legendre polynomials. (An, CO 2)

20. For the following linear system , find the general solution , differential equation of the paths and its solution. Sketch a few paths showing the direction of increasing t and discuss the stability of the critical point (0,0). (A, CO 3)
dx/dt = ax-y
dy/dt = x+ay

21. State and prove Picard's theorem. (An, CO 4)

22. Explain the problem of vibrating srting. (An, CO 1)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Summarize the concepts of Sturm Separation theorem and Sturm Liouville problems	A	1, 6, 11, 15, 22	11
CO 2	Explain the properties of Legendre and Bessel's polynomials	A	3, 7, 9, 12, 16, 19	12
CO 3	Analyze the concept of linear and nonlinear systems and their stability	An	2, 8, 10, 13, 18, 20	12
CO 4	Illustrate the ideas of existence and uniqueness of solutions	An	4, 14, 17, 21	10

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;