Reg. No .... Name **23U362**

B. Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 3 : MATHEMATICS (CORE COURSE) FOR B.Sc COMPUTER APPLICATION

COURSE : 19U3CRCMT4 : VECTOR CALCULUS, TRIGONOMETRY AND MATRICES

(For Regular - 2022 Admission and Improvement/Supplementary - 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. If $\phi = 2xz^4 - x^2y\ ,\ find\ grad\ \phi.$
2. Evaluate $div\left[(xy\sin z)\boldsymbol{i} + \left(y^2\sin x\right)\boldsymbol{j} + \left(z^2\sin(xy)\right)\boldsymbol{k}\right]$ at the point $\left(0, \frac{\pi}{2}, \frac{\pi}{2}\right)$.
3. Show that $\overrightarrow{\mathrm{F}}$ = $yz\ \ i$ +$zx\ \ j$ +$xy\ k$ is irrotational.
4. State Guass's divergence theorem.
5. **A vector field is given by F=(siny)i+x(1+cosy)j.Evaluate the line integral over the circular path given by $x^2+y^2=a^2$,z=0.**
6. If **F**=$(3x^2+6y)$**i**-14yz**j**+$20xz^2$**k**,evaluate the line integral $\int_C F.\,dr$ from (0,0,0) to (1,1,1) along the path x=t,$y=t^2$,$z=t^3$.
7. Prove that sinh (x+y) = sinh x cosh y + cosh x sinh y.
8. Separate into real and imaginary parts the expression tan(x+iy).
9. Prove that $tanh(x+y)$ = $\dfrac{tanhx + tanhy}{1 + tanhxtanhy}$
10. Find the rank of $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$
11. Prove that $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.
12. Find the eigen values of the matrix $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. In what direction from(3,1,-2)is the directional derivative of $\phi=x^2y^2z^4$ maximum and what is its magnitude?
14. If A is a vector function and ϕ is a scalar function,then prove that div(ϕA) =ϕ divA + (gradϕ).A.
15. If F=2yi-zj+xk, evaluate $\oint_c F \times dr$ along the curve x = cost,y=sin t,z=2 cos t from t=0 to t=$\frac{\pi}{2}$
16. Evaluate $\iint_S \boldsymbol{f}.\,\widehat{n}\ dS\ where\ \boldsymbol{f} = z\boldsymbol{i} + x\boldsymbol{j} - y^2z\boldsymbol{k}$ and S is the surface of the cylinder $x^2 + y^2 = 1$ included in the first octant between planes z=0 and z=2.

17. If $\sin\ (\theta\ + i\phi) = \tan\ (x + iy)$, show that $\frac{\tan\ \theta}{\tan h\ \phi} = \frac{\sin\ 2x}{sinh\ 2y}$

18. Factorise x^6+1 into real factors.

19. Solve the system of equations by matrix method.
$x + y + z = 8, \quad x - y + 2z = 6, \quad 3x + 5y - 7z = 14.$

20. Find the inverse of A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by elementary row operations.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. If **A** is a vector function and φ is a scalar function,then prove that
a)div(φ**A**) =φ div**A** + (gradφ).**A**
b)curl(φ**A**) =φ curl**A** + (gradφ)x**A**

22. Verify Green's theorem in the plane for $\oint_C \ (3x^2 - 8y^2)dx \ + \ (4y - 6xy)dy \ where\ C$ is the boundary of the region defined by x=0,y=0,x+y=1.

23. Find the sum to infinity of the following series.
a) $\sin\alpha.\cos\alpha + \sin^2\alpha.\cos2\alpha + \sin^3\alpha.\cos3\alpha + \ldots\ldots\ldots\infty.$
b) $\sin\ \alpha - \frac{\sin\ (\alpha+2\beta)}{2!} + \frac{\sin\ (\alpha+4\beta)}{4!} - \ldots\ldots\ldots\ldots\ldots\ldots\infty$

24. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(10 x 3 = 30)