Reg. No .... Name **23U357**

B.Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 3 :MATHEMATICS (COMPLEMENTARY FOR PHYSICS/CHEMISTRY)

COURSE : 19U3CPMAT3 : DIFFERENTIAL EQUATIONS, MATRICES AND TRIGONOMETRY

(For Regular 2022 Admission and Improvement / Supplementary 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Prove that $\sin^5 x = \frac{1}{16}[\sin 5x - 5\sin 3x + 10\sin x]$.
2. Define the order of a differential equation.
3. Explain the term normal form with examples.
4. Find one of the solution of $x(y-z)p + y(z-x)q = z(x-y)$.
5. State true or false:if $u_{1,} u_2$,.............u_n are n independent solution,then $c_1u_1 + c_2u_2 + \ldots\ldots\ldots\ldots c_nu_n$ is also a solution.
6. State the condition for consistency of a system of equations.
7. Solve $(x+y)dx - xdy = 0$.
8. Find the integrating factor of the linear equation $(x^2+1)\frac{dy}{dx}+4xy=x$.
9. Form a partial differential equation from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ by eliminating the arbitrary constants.
10. Find characteristic equation of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 8 & -1 \\ -1 & -2 & 6 \end{bmatrix}$
11. Expand $\sin^7 x$ in a series of sine of multiples of x.
12. Evaluate the eigen values of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. find the eigne values of $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ and evaluate the eigen vector corresponding to the larges eigen value.
14. Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{bmatrix}$
15. Solve the linear equation $\left(x^2+1\right)\frac{dy}{dx} + 4xy = x$ with initial condition $y\left(2\right) = 1$
16. Solve the differential equation $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$

17. Show that
$\cos 8\theta = \cos^8\theta - 28\cos^6\theta\sin^2\theta + 70\cos^4\theta\sin^4\theta - 28\cos^2\theta\sin^6\theta + \sin^8\theta.$

18. Sum the series $\frac{5\cos\theta}{1} + \frac{7\cos 3\theta}{3!} + \frac{9\cos 5\theta}{5!} + \cdots$inf.

19. Form a partial differential equation by eliminating the arbitrary function from $z = f(x+it) + g(x-it);\ i = \sqrt{-1}.$

20. Verify that u=coskx sinky is a solution of laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. (a) Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x\ \sin y$, given that $\frac{\partial z}{\partial y} = -2\sin y$ when x=0 and z=0 when y is an odd multiple of $\frac{\pi}{2}$
(b)form a PDE by eliminating the arbitrary constants from $z = ax + by + a^2 + b^2$.

22. Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$.

23. Verify cayley Hamilton theorem for the matrix A and find A^{-1}. A = $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.

24. Sum the series $1 + c\cos\alpha + c^2\cos 2\alpha + c^3\cos 3\alpha + \cdots$, where c is less than unity and sum the series $c\sin\alpha + c^2\sin 2\alpha + c^3\sin 3\alpha + \cdots$, where c is less than unity.

(10 x 3 = 30)