

B. Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023
SEMESTER 3 : STATISTICS (FOR MATHEMATICS AND COMPUTER APPLICATION)

COURSE : 19U3CPSTA03 / 19U3CRCST03 : PROBABILITY DISTRIBUTIONS

(For Regular 2022 Admission and Improvement / Supplementary 2021/2020/2019 Admissions)

Time : Three Hours

Max. Marks: 75

(Use of Scientific Calculator and Statistical tables are permitted)

PART A

Each question carries 1 mark. Maximum marks from this part is 10

1. What is Cauchy-Schwartz inequality?
2. What is the distribution of ratio of two independent random variables following Gamma distribution?
3. If $V(X) = 1$ then find $V(2X + 3)$
4. What do you mean by convergence in probability?
5. If X is a random variable with $E(X) = 3$, $V(x) = 2$, then find k if $P[|X-3| < 2] \geq k$
6. If X is a Bernoulli random variable then find the distribution of X^2 ?
7. Define lack of memory property of exponential distribution?
8. Let X and Y be two independent chi-square variates with degrees of freedom 2 and 3 respectively. Find the distribution of $3X/2Y$
9. What is the square of a random variable following t distribution with n degrees of freedom?
10. If X and Y are two independent random variables with standard deviations 3 and 2 respectively, find the variance of $2X-3Y$
11. If $X_1, X_2, X_3, \dots, X_{16}$ is a random sample from $N(20,8)$, then find the standard error of the sample mean ?
12. For a Binomial distribution the mean is 6 and standard deviation is $\sqrt{2}$. Write the distribution function.

PART B

Each question carries 3 marks. Maximum marks from this part is 15

13. The mean and variance of a binomial variate X with parameter n and p are 16 and 8. Find (1) $P(x=0)$ (2) $P(X=1)$ (3) $P(X \geq 2)$
14. Define characteristic function? Give its properties?
15. Find the mean of gamma distribution with parameter 'p'.
16. Find the quartiles of a normal random variable with mean 25 and s.d. 2 ?
17. Define mathematical expectation? Derive multiplication theorem on expectation?
18. Define chi-square statistic? Write the p.d.f. of chi-square distribution?
19. Use Bernoulli's theorem to find the least number of tosses of a fair coin required in order that the probability will be at least 0.95 that the frequency ratio of the number of heads will lie between 0.35 and 0.65.

PART C

Each question carries 5 marks. Maximum marks from this part is 20

20. Find the M.G.F. of the random variable X whose probability function $P(X=x) = 1/2^x$; $x=1,2,3, \dots$. Hence find its mean?
21. Explain the importance of normal distribution in Statistics?
22. Show that for a normal distribution with mean μ , and standard deviation σ , $\mu_{2r} = 1 \cdot 3 \cdot 5 \cdot \dots \cdot (2r-1) \sigma^{2r}$
23. State and prove Lindberg -Levy form of Central limit theorem?
24. State the interrelation among Normal, Chi square 't' and 'F' distribution
25. In a certain factory turning out optical lenses there is a small chance of one out of 1500 for any one lens to be defective. The lenses are supplied in packets of 4500 each. Use Poisson distribution to calculate the approximate number of packets containing one defective lens in a consignment of 20,000 packets

PART D

(Each question carries 10 marks. Maximum marks from this part is 30)

26. Define normal distribution. Explain the characteristics of normal distribution. Find mean and variance.
27. Two random variables X and Y have the following joint p.d.f. $f(X,Y) = 2 - X - Y$, $0 \leq X \leq 1$, $0 \leq Y \leq 1$ and zero elsewhere find (I) $E(X)$ and $E(Y)$ (II) $V(X)$ and $V(Y)$ (III) $Cov(X,Y)$ and correlation coefficient
28. State Tchebycheff's inequality. A random variable takes values $-1,1,3,5$ with probabilities $1/6, 1/6, 1/6, 1/2$ respectively. Find by direct computation $P[|x-3| \geq 1]$ Find an upper bound to this probability by applying Tchebycheff's inequality?
29. Define (1) Statistic (2) Sampling distribution (3) Standard error. Also derive the sampling distribution of the sample mean drawn from a normal population.