Reg. No .... Name **23U326**

B.Sc DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 3 : MATHEMATICS FOR B.Sc. COMPUTER APPLICATIONS

COURSE : **19U3CRCMT3 : CALCULUS**

(For Regular - 2022 Admission and Improvement/Supplementary - 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Evaluate $\iint_R y\,dy\,dx$, where R is the region bounded by the parabolas $y^2 = 4x \; and \; x^2 = 4y.$
2. Evaluate the integral $\int_0^{\pi} \sin^2\left(1+\frac{\theta}{2}\right)d\theta.$
3. Evaluate $\int_0^3 \int_0^2 xy\left(x+y\right)\,dy\,dx.$
4. Evaluate the integral $\int_0^{\pi/2} \frac{3\ \sin x\ \cos x}{\sqrt{1+3\sin^2 x}}dx.$
5. Find the points of inflecion on the curve $y = x^4 - 6x^2 + 8x - 1.$
6. Find $f_x, f_y \; and \; f_z \; if \;\; f\left(x,\ y,\ z\right) = \sin^{-1}\left(xyz\right).$
7. $Find\ the\ n^{th}\ derivative\ of\ \sin\ x\ \cos\ 3x.$
8. Expand $f\left(x\right) = 2x^3 + 7x^2 + x - 6 \; in \; powers \; of \; \left(x-2\right).$
9. Find the values of $\frac{\partial f}{\partial x} and \; \frac{\partial f}{\partial y}$ at the point (2,-1) if $f\left(x,y\right) = 3x^3y + 4xy^2 - 2x + 4y - 5.$
10. Use the chain rule to find the derivative of $w = x^2 + y^2$ with respect to t along the path x = cos t , y = sin t.What is the derivative's value at $t = \pi$.
11. $If \; f\left(x,y\right) = x^2y - 2xy \; and \;\; R:\ 0 \le x \le 3, \quad -2 \le y \le 0, \; then \; evaluate \;\; \iint_R f\left(x,y\right)dA.$
12. Find the centroid of the region R between the semi-circle $y = \sqrt{a^2 - x^2}$ and the x-axis.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Find the volume of the solid generated by revolving the region bounded by the curve y=x^2 and the lines y=0,x=2 about the x-axis.
14. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and by the lines y = 2, and x = 0 about the line y = 2.
15. $Show\ that\ the\ n^{th} derivative\ of\ y = \tan^{-1} x\ is$

    $\left(-1\right)^{n-1}\left(n-1\right)!\sin\ n\left(\frac{\pi}{2} - y\right)\ \sin^n\left(\frac{\pi}{2} - y\right).$
16. Using chain rule express $\frac{\partial w}{\partial r} and \; \frac{\partial w}{\partial \theta}$ in terms of $r \; and \; \theta$, if $w = \tan^{-1}\left(y/x\right),\ x = r\ \cos\theta,\ y = r\ \sin\theta.$ Also evaluate $\frac{\partial w}{\partial r} and \; \frac{\partial w}{\partial \theta}$ at the point (1, π/6).
17. Verify that $\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial y \partial x}$, when $w = x^y + \sin\left(xy\right).$
18. Evaluate $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\,dz\,dy\,dx.$
19. Change the cartesian integral into equivalent polar integral and hence evaluate $\iint_R \left(x-y\right)^4 e^{x+y}\,dx\,dy$, where R is the square with vertices (1, 0), (2, 1), (1, 2) and (0, 1).
20. Find the radius of curvature of the cardioid $r = a(1 - \cos\ \theta).$

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

22. Using Lagrange multipliers, find the greatest and smallest values that the function f(x,y)=xy takes on the ellipse $x^2 + 2y^2 = 1$.

23. a) Find all asymptotes of the curve $y^3 - x^2y + 2y^2 + 4y + x = 0$.
b) Show that the envelope of a circle whose centre lies on the parabola $y^2 = 4ax$ and which passes through its vertex is $2ay^2 + x\left(x^2 + y^2\right) = 0$.

24. a) Find the area of the surface generated by revolving the curve$y = \sqrt{2x+1}, \quad 0 \leq x \leq 3,$ about the x-axis.
b) Find the volume of the solid generated by revolving the region bounded by the x-axis,the curve y = $3x^4$ and the lines x = 1 and x = -1 about the line y = 3.

(10 x 3 = 30)