Reg. No .... Name **23U325**

B.Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 3 : MATHEMATICS

COURSE : **19U3CRMAT03 : VECTOR CALCULUS, THEORY OF EQUATIONS AND MATRICES**

(For Regular - 2022 Admission and Improvement/Supplementary - 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 8 & -1 \\ -1 & -2 & 6 \end{bmatrix}$.
2. Find the unit vector normal to the surface $i) xy^3z^2 = 4\ at\ (-1,-1,2)$
   $ii) x^2y + 2xz = 4\ at\ (2,-2,3)$
3. Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.
4. Find the inverse of $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$.
5. Evaluate the eigen values of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.
6. Find the sum of the cubes of the roots of the equation $x^3 - 2x^2 + x + 1 = 0$
7. State Descarte's rule of signs and apply it to prove that the equation $x^3 + 2x + 3 = 0$ has one negative and two imaginary roots.
8. Show that $\nabla.(\overline{A} \times \overline{B}) = \overline{B}.(\nabla \times \overline{A})$- $\overline{A}.(\nabla \times \overline{B})$.
9. State Gauss divergence theorem and prove that for any closed surface S, $\iint_s curl\ \bar{F}.\hat{n}ds = 0$.
10. If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, find the value of $\sum \alpha^2$.
11. Explain the term normal form of a matrix with examples.
12. State Stoke's theorem.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Solve $x^4 - 4x^3 + 7x^2 - 6x + 2 = 0$ using Ferrari's method.
14. Evaluate $\iint_s (xi + yi + z^2k).\hat{n}ds$,where S is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane z=1.
15. Evaluate grad ϕ when $\phi = 3x^2y - y^3z^2$ at the point (1,-2,-1)
16. Use divergence theorem to evaluate $\iint_s \overline{F}.ds$ where $\overline{F} = x^3i + y^3j + z^3k$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.
17. Prove that $\nabla.(\overline{A}.\overline{B}) = \nabla.\overline{A} + \nabla.\overline{B}$.

18. Evaluate the rank of the matrix $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix}$.

19. Verify Cayley Hamilton theorem for A = $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

20. Find the roots of the equation $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Solve $x^5 - 10x^2 + 15x - 6 = 0$ given that it has multiple roots.

22. Verify divergence theorem for $\overline{F} = 4xi - 2y^2j + z^2k$ taken over the region bounded by the cyclinder $x^2 + y^2 = 4, z = 0 \; and \; z = 3$.

23. Calculate the eigen values and eigen vector of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

24. a) Prove that $\nabla^2 (r^n) = n(n+1)r^{n-2}$
b)If $\bar{v} = 3x^2y^2z^4i + 2x^3yz^4j + 4x^3y^2z^3k \; show \; that \; \bar{v}$ is a conservative field and find its scalar potential.

(10 x 3 = 30)