Reg. No .................... Name .............................. **23P132**

## M.Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

### SEMESTER 1 : MATHEMATICS

### COURSE : **21P1MATT03 : REAL ANALYSIS**

*(For Regular - 2023 Admission and Improvement/Supplementary -2022/2021 Admissions)*

Duration : Three Hours Max. Weights: 30

**PART A**

**Answer any 8 questions** **Weight: 1**

1. Prove that $\int_{a_-}^{b} f d\alpha \leq \int_{a}^{b^-} f d\alpha$ (U)
2. What is the Riemann-Stieltjes integral, and how does it differ from the Riemann integral? (U)
3. Show by an example that a function of bounded variation need not be continuous. (E, CO 1)
4. Find the radius of convergence of the series $x + \frac{x^2}{2^2} + \frac{2!}{3^2}x^3 + \frac{3!}{4^4}x^4 + \ldots\ldots..$ (A)
5. Construct a continous function which is not of bounded variation. (A, CO 1)
6. If $f_1, f_2 \in \mathscr{R}(\alpha)$ on [a,b], then prove that $f_1 + f_2 \in \mathscr{R}(\alpha)$ on $[a, b]$. (U, CO 2)
7. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded. (An, CO 1, CO 2)
8. Prove that $\lim_{x\to 0}(1 + x)^{1/x} = e$. (A, CO 4)
9. If $f$ is continuous on$[a, b]$ and if $f'$ exists and is bounded in the interier, say |f'(x)|≤A for all $x$ in $(a, b)$, then prove that f is of bounded variation on [a,b] (U, CO 1)
10. If $\{f_n\}$ and $\{g_n\}$ converges uniformly on a E, prove that $\{f_n + g_n\}$ converges uniformly on E. (A)

**(1 x 8 = 8)**

**PART B**

**Answer any 6 questions** **Weights: 2**

11. Determine the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{2n!}{(n!)^2} z^n$ (A, CO 4)
12. If $f$ is continuous on [a,b], then prove that $f \in \mathscr{R}(\alpha)$. (A, CO 1)
13. If $f$ is a function defined on $[a, b]$, prove that $f$ is of bounded variation on $[a, b]$ if and only if $f$ can be expressed as the difference of two increasing functions. (R, CO 1)
14. If f is monotonic on [a, b ], and if α is continuous on [a, b ], then prove that f∈R(α) (R, CO 2)
15. Find the total variation of the function $f(x) = \sin 2x$ over the interval $[0, 2\pi]$. (An, CO 1)
16. Let $\alpha$ be monotonically increasing on [a,b]. Suppose $f_n \in \mathfrak{R}(\alpha)$ on [a,b], for n=1,2,.....and suppose $f_n \to f$ uniformly on [a,b]. Then prove that $f \in \mathfrak{R}(\alpha)$ on [a,b] and $\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$ (R)

17. Discuss the uniform convergece of $f_n$, where $f_n(x) = \frac{1}{1+nx^2}$ for all $x \in [0, \infty)$ (A)

18. Derive a relation between the Riemann-Stieltjes upper sums of a function corresponding to a partition and its refinement? (E, CO 2)

**(2 x 6 = 12)**

**PART C**

**Answer any 2 questions** **Weights: 5**

19. Prove that the series $\sum \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval, but does not converge absolutely for any value of $x$. (U, CO 3)

20. a) Prove that every absolutely continuous function on [a,b] is continuous and of bounded variation on [a,b]
b)Prove that $f$ is absolutely continuous if it satisfies a uniform Lispschitz condition of order 1 on [a,b] (R, CO 1)

21. Suppose $f(x)f(y) = f(x+y)$ for all real $x$ and $y$. Assuming $f$ is continuous, prove that $f(x) = e^{cx}$, where c is constant. (A, CO 4)

22. Assume $\alpha$ increases monotonically and $\alpha' \in \mathfrak{R}$. Let $f$ be a bounded real function on [a,b], then prove that $f \in \mathfrak{R}(\alpha)$ if and only if $f\alpha' \in \mathfrak{R}$, In that case show that $\int_a^b f d\alpha = \int_a^b f\left(x\right)\alpha'\left(x\right)dx$ (R)

**(5 x 2 = 10)**

OBE: Questions to Course Outcome Mapping

| CO | Course Outcome Description | CL | Questions | Total Wt. |
|---|---|---|---|---|
| CO 1 | Explain the functions of bounded variations, rectifiable curves, paths and equivalence of paths. | U | 3, 5, 7, 9, 12, 13, 15, 20 | 15 |
| CO 2 | Illustrate the properties of Riemann-Stieljes integral. | An | 6, 7, 14, 18 | 6 |
| CO 3 | Analyze the uniform convergence of a sequence of functions with continuity, integrability, differentiability. | An | 19 | 5 |
| CO 4 | Apply the properties of power series to exponential, logarithmic and trigonometric functions. | A | 8, 11, 21 | 8 |

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;