Reg. No .... Name **23P118**

M. Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 1 : MATHEMATICS

COURSE : **21P1MATT02 : ALGEBRA**

(For Regular - 2023 Admission and Improvement/Supplementary -2022/2021 Admissions)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	True or False: $\mathbb{R}$ is not perfect. Justify your answer.	(U, CO 4)
2.	Find all $c \in \mathbb{Z}_3$ such that $\mathbb{Z}_3[x]/ < x^3 + x^2 + c >$ is a field?	(A, CO 2)
3.	How many polynomials (including the zero polynomial) are there of degree ≤ 2 in $\mathbb{Z}_5[x]$?	(A, CO 2)
4.	Is $\mathbb{R}(i)$ the smallest subfield of $\mathbb{C}$ containing a zero of $x^2 + 1$? Justify your answer.	(U, CO 3)
5.	Show that $1 + \sqrt{2}$ is an algebraic number.	(A, CO 3)
6.	Consider the evaluation homomorphism $\phi_5 : \mathbb{Q}[x] \to \mathbb{R}$. Find six elements in the kernel of the homomophism ϕ_5.	(A, CO 2)
7.	Define simple extension and algebraic extension of a field. Is every finite extension an algebraic extension?	(U, CO 3)
8.	What is the order of $G(\mathbb{Q}(\sqrt[3]{2}), i\sqrt{3})/\mathbb{Q}(\sqrt[3]{2}))$?	(An, CO 4)
9.	What are the possible numbers of Sylow 2-subgroups of a group of order 24?	(R, CO 1)
10.	Is every group of order 102 simple? Justify your answer.	(An, CO 1)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	Find the degree and a basis of $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ over $\mathbb{Q}$.	(A, CO 3)
12.	Find the splitting field of $x^3 - 2$ over $\mathbb{Q}$.	(E, CO 4)
13.	Show that a group of order 48 is not simple.	(E, CO 1)
14.	Show that $x^4 - 22x + 1$ is irreducible over $\mathbb{Q}$.	(E, CO 2)
15.	For the evaluation homomorphism $\phi_5 : \mathbb{Z}_7[x] \to \mathbb{Z}_7$, evaluate $\phi_5(3x^{106} + 5x^{99} + 2x^{53})$.	(E, CO 2)
16.	Show that $\mathbb{Q}(2^{1/2}, 2^{1/3}) = \mathbb{Q}(2^{1/6})$	(E, CO 3)
17.	Show that if $\alpha, \beta \in \overline{F}$ are both separable over F, then $\alpha \pm \beta$, $\alpha\beta$, and α/β, if $\beta \neq 0$, are all separable over F.	(An, CO 4)
18.	(a) State the fundamental theorem of Finitely generated abelian groups. (b) Find all abelian groups upto isomorphism of order 720.	(A, CO 1)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. (a). Show that if F is a field of prime characteristic p with algebraic closure $\overline{F}$, then $x^{p^n} - x$ has p^n distinct zeroes in $\overline{F}$.
(b). Show that if F is a field of prime characteristic p, then $(\alpha + \beta)^{p^n} = \alpha^{p^n} + \beta^{p^n}$ for all $\alpha, \beta \in F$ and all positive integers n. (An, CO 3)

20. Show that every finite field is perfect. (E, CO 4)

21. (a). Let X be a G-set. For $x_1, x_2 \in G$, let $x_1 \sim x_2$ if and only if there exists $g \in G$ such that $gx_1 = x_2$. Show that $\sim$ is an equivalence relation on X. What is the equivalence class of $x \in X$ under this equivalence relation known as?
(b). Let X be a G-set. Show that $G_x = \{g \in G \mid gx = x\}$ is a subgroup of G for each $x \in X$. What is this subgroup known as? (E, CO 1)
(c). Let G be a group of order p^n and let X be a finite G-set. Let $X_G = \{x \in X \mid gx = x \text{ for all } g \in G\}$. Show that $|X| \cong |X_G| (\bmod\ p)$.

22. (a). State and prove Eisenstein's criterion for irreducibility over $\mathbb{Q}$.
(b). Show that the p^{th} cyclotomic polynomial is irreducible over $\mathbb{Q}$ for any prime p. (E, CO 2)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Develod ideas of finitely generated abelian groups, Sylow theorems and applications.	E	9, 10, 13, 18, 21	11
CO 2	Explain the concept of rings of polynomials, factorisation of polynomials and ideal structure.	E	2, 3, 6, 14, 15, 22	12
CO 3	Illustrate the idea of extension fields, algebraic extensions and geometric constructions.	E	4, 5, 7, 11, 16, 19	12
CO 4	Devrlop ideas of automorphisms of fields, isomorphism extension theorem and Galois theory.	E	1, 8, 12, 17, 20	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;