Reg. No .... Name **23U160**

B.Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 1 : COMPLEMENTARY MATHEMATICS FOR B.Sc. PHYSICS/CHEMISTRY

COURSE : **19U1CPMAT1 : CALCULUS-1**

(For Regular 2023 Admission and Improvement/Supplementary 2022/2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Define local maximum and local minimum of a function $f(x, y)$ in a region R.
2. A pyramid $3\,m$ high has a square base that is $3\,m$ on a side. The cross-section of the pyramid perpendicular to the altitude $x\,m$ down from the vertex is a square $x\,m$ on a side. Find the volume of the pyramid.
3. State first derivative test for monotonic functions.
4. Evaluate f_x,f_y, and f_z if $f(x, y, z) = e^{-(xyz)}$.
5. Verify Rolle's theorem for the function $f(x) = \frac{x^3}{3} - 3x$ in the interval $[-3, 3]$
6. State the stronger form of Fubini's theorem.
7. Evaluate f_x and f_y if $f(x, y) = \tan^{-1}(y/x)$.
8. Find the volume of the torus (doughnut) generated by revolving a circular disk of radius a about an axis in its plane at a distance $b \geq a$ from its center.
9. Calculate $\int\int f(x, y)dA$ for $f(x, y) = 1 - 6x^2y$ and $R : 0 \leq x \leq 2,\ -1 \leq y \leq 1$.
10. Verify mean value theorem for the function $f(x) = x^2 + 2x - 1$, in the interval $[0, 1]$
11. Let f be continuous on the symmetric interval $[-a, a]$. Show that $\int_{-a}^{a} f(x)dx = 0$, if f is odd.
12. By converting to an equivalent polar integral integrate $f(x, y) = \frac{\ln(x^2 + y^2)}{\sqrt{x^2 + y^2}}$ over the region $1 \leq x^2 + y^2 \leq e$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Find the lateral (side) surface area of the cone generated by revolving the line segment $y = x/2,\ 0 \leq y \leq 4$, about the x-axis.
14. Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the intervals on which f is increasing and decreasing..
15. Find the volume of the solid generated by revolving each region enclosed by the triangle with vertices $(1, 0)$, $(2, 1)$, and $(1, 1)$ about the y-axis.
16. Verify that $w_{xy} = w_{yx}$ for the function $w = \ln(2x + 3y)$.
17. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$ by integrating with respect to y.

18. Find the limits of integration for evaluating the triple integral of a function $F(x, y, z) = 1$ over the tetrahedron D with vertices $(0, 0, 0), (1, 1, 0), (0, 1, 0), and (0, 1, 1)$.

19. Find the extreme values of $V(x) = x(10 - 2x)(16 - 2x), 0 < x < 5$.

20. Find all the local maxima, local minima, and saddle points of the function $f(x, y) = \frac{1}{x^2+y^2-1}$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Use the theorems of Pappus to find the lateral surface area and the volume of a right circular cone.

22. Find the volume of the solid whose base is the region in the xy - plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $z = x + 4$.

23. Find the extreme values of $f(x, y) = xy$ subject to the constraint $g(x, y) = x^2 + y^2 - 10 = 0$.

24. State and prove Mean Value Theorem.

(10 x 3 = 30)