

**B. Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023****SEMESTER 1 : MATHEMATICS (COMMON FOR BSC CA/BCA)****COURSE : 19U1CRCMT1/19U1CPCMT1 : FOUNDATION OF MATHEMATICS***(For Regular 2023 Admission and Improvement/Supplementary 2022/2021/2020/2019 Admissions)*

Time : Three Hours

Max. Marks: 75

**PART A****Answer any 10 (2 marks each)**

1. What is the value of  $x$  after each of these statements is encountered in a computer program, if  $x = 1$  before the statement is reached?
  - a) if  $x + 2 = 3$  then  $x := x + 1$
  - b) if  $(x + 1 = 3)$  OR  $(2x + 2 = 3)$  then  $x := x + 1$
2. If  $U = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$ ,  $A = \{4, 6, 10, 12, 18\}$  and  $B = \{4, 8, 12, 16, 20\}$ . Find the bit strings representing  $A$  and  $A \cup B$ .
3. Express the statements "Some student in this class has visited Mexico" and "Every student in this class has visited either Canada or Mexico" using predicates and quantifiers.
4.
  - a) Define a lattice.
  - b) Give an example of a poset with five elements that is a lattice and an example of a poset with five elements that is not a lattice.
5. If  $A = \{a, b, c\}$  and  $B = \{1, 2, 3, 4\}$ . Find the number of functions from  $A$  into  $B$  and  $B$  into  $A$ .
6. Draw the graph of the polynomial function  $f(x) = x^2 - 2x - 3$ .
7. Find  $\sum_{j \in S} j^2$  and  $\sum_{j \in S} j$ , where  $S = \{1, 3, 5, 7\}$ .
8. Find the number of relations from  $A = \{a, b, c, d\}$  to  $B = \{x, y\}$ .
9. Verify that 496 is a perfect number.
10. What is the composite of the relations  $R$  and  $S$ , where  $R$  is the relation from  $\{1, 2, 3\}$  to  $\{1, 2, 3, 4\}$  with  $R = \{(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)\}$  and  $S$  is the relation from  $\{1, 2, 3, 4\}$  to  $\{0, 1, 2\}$  with  $S = \{(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)\}$ ?
11. For any three integers,  $a, b, c$ , prove If  $a | b$  and  $a | c$ , then  $a | (bp + cq)$ , for all integer values of  $p$  and  $q$ .
12. Let  $A = \{a_1, a_2, a_3\}$  and  $B = \{b_1, b_2, b_3, b_4, b_5\}$ . Which ordered pairs are in the relation  $R$  represented by the matrix  $M_R$ 

$$\begin{matrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{matrix}$$

**(2 x 10 = 20)****PART B****Answer any 5 (5 marks each)**

13. Let  $A = \{a, b, c, d, e\}$ ,  $B = \{1, 2, 3, 4, 5\}$ ,  $C = \{0, 4, 7, 8\}$ . Consider the relations  $R$  from  $A$  to  $B$  and the relation  $S$  from  $B$  to  $C$ , defined by  $R = \{(a, 2), (a, 5), (b, 3), (c, 1), (c, 3), (d, 5)\}$  and  $S = \{(1, 7), (2, 0), (2, 7), (5, 0), (5, 4)\}$ . Find the matrices  $M_R, M_S$  and  $M_{S \circ R}$ .
14. What are the terms  $a_0, a_1, a_2, a_3$  of the  $\{a_n\}$  where  $a_n = 2^n + (-2)^n$
15. Show that set of all integers is countable.
16. Show that  $n(n+1)(2n+1)$  is divisible by 6.

17. For any sets, Prove that  $A - B = A \cap \overline{B}$
18. Show that there exist irrational numbers  $x$  and  $y$  such that  $xy$  is rational.
19. Determine whether the relation  $R$  on the set reflexive, symmetric, antisymmetric and transitive, where of all Web pages is  $(a, b) \in R$  if and only if
- everyone who has visited Web page  $a$  has also visited Web page  $b$ .
  - there are no common links found on both Web page  $a$  and Web page  $b$ .
  - there is at least one common link on both Web page  $a$  and Web page  $b$ .
  - there is a Web page that includes links to both Web page  $a$  and Web page  $b$ .
20. a) Define a partial ordering.  
b) Show that the divisibility relation on the set of positive integers is a partial order. **(5 x 5 = 25)**

### PART C

**Answer any 3 (10 marks each)**

21. Construct a truth table for each of these compound propositions.
- $(p \vee q) \rightarrow (p \oplus q)$
  - $(p \oplus q) \rightarrow (p \wedge q)$
  - $(p \vee q) \oplus (p \wedge q)$
  - $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
  - $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
22. a) Define the floor and ceiling functions and display the graphs of these functions.  
b) Prove that if  $x$  is a real number, then  $[2x] = [x] + [x + 1/2]$ .
23. Draw the Hasse diagram for the partial ordering  $\{(A, B) \mid A \subseteq B\}$  on the power set  $P(S)$  where  $S = \{a, b, c\}$ .
24. Prove that  $3^{2n+1} + 2$  is divisible by 7. **(10 x 3 = 30)**