

B. Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023**SEMESTER 1 : MATHEMATICS****COURSE : 19U1CRMAT1 : CALCULUS***(For Regular 2023 Admission and Improvement/Supplementary 2022/2021/2020/2019 Admissions)*

Time : Three Hours

Max. Marks: 75

PART A**Answer any 10 (2 marks each)**

1. Find the open intervals on which $f(x) = x^2 - 3x + 8$ is decreasing
2. Find f_x and f_y for $f(x, y) = 2x^2y^3 + 2y + 4x$
3. Describe the level surfaces of $f(x, y, z) = z^2 - x^2 - y^2$.
4. Find the open intervals on which $f(x) = 5 - 4x - x^2$ is increasing
5. Find the formula for the volume of a sphere of radius r .
6. Find the area of the region enclosed by the curves $y = x^2$, $y = \sqrt{x}$, $x = \frac{1}{4}$ and $x = 1$.
7. Find the domain and range of $\sinh x$.
8. Prove that $\cosh^2 x - \sinh^2 x = 1$.
9. Find the area of the region enclosed by the curves $x = \frac{1}{y}$, $x = 0$, $y = 1$ and $y = e$.
10. Find all critical points of $f(x) = 3x^{5/3} - 15x^{2/3}$.
11. Define level curves with an example.
12. Find $\lim_{x \rightarrow \frac{\pi}{4}} (1 - \tan x) \sec 2x$.

(2 x 10 = 20)**PART B****Answer any 5 (5 marks each)**

13. Sketch the region enclosed by the curves and find its area $y = x$, $y = 4x$, $y = -x + 2$.
14. Find the absolute maximum and minimum values of $f(x) = 4x^2 - 12x + 10$ on the interval $[1, 2]$.
15. Determine $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx}$
16. Evaluate $\int \sinh^6 x \cosh x \, dx$ and $\int \sqrt{\tan hx} \operatorname{sech}^2 x \, dx$.
17. Find the exact arc length of the curve $y = 3x^{3/2} - 1$ over the interval $[0, 1]$.
18. Let $z = f(x, y, z)$. Show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.
19. Find the absolute extrema of $f(x) = 6x^{4/3} - 3x^{1/3}$ on the interval $[-1, 1]$ and determine where these values occur.
20. Find the area of the surface generated by revolving $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$ about x-axis.

(5 x 5 = 25)**PART C****Answer any 3 (10 marks each)**

21. State Rolle's theorem. It is given that the Rolle's theorem holds function the function $f(x) = x^3 + bx^2 + cx$, $1 \leq x \leq 2$ at the point $x = \frac{4}{3}$. find the value of b and c .

22. Find the points on the sphere $x^2 + y^2 + z^2 = 36$ that is closest to and farthest from the point $(1, 2, 2)$.
23. Find the arclength of the curve $x = \cos t + t \sin t, y = \sin t - t \cos t, 0 \leq t \leq \pi$.
24. Sketch the graph of curve $y = \frac{(x-1)(x-3)}{x^2}$.

(10 x 3 = 30)