Reg. No .... Name **23P364**

M. Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT15 : MULTIVARIATE CALCULUS**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. State Convolution theorem for Fourier transforms. (A, CO 1)
2. Let $f : R^2 \to R^3$ be defined by the equation $f(x, y) = (\sin x \cos y, \sin x \sin y, \cos x \cos y)$. Determine the Jacobian matrix. (A, CO 2)
3. Find the Laplace transform of $f(x) = \cos ax$ (A, CO 1)
4. Define basic k-forms. (U, CO 4)
5. Define the term primitive mapping. (An, CO 4)
6. Define saddle point. (R, CO 3)
7. State the exponential form of Fourier series. (A, CO 1)
8. Find the number of basic k-forms in $\mathbb{R}^n$. (U, CO 4)
9. Show that the components of $f'(c)$ are the dot product of the successive rows of the Jacobian matrix with the vector v. (A, CO 2)
10. Define stationary points. (R, CO 3)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Show that $\int_{T\phi} \omega = \int_{\phi} \omega_T$. (An, CO 4)
12. If f is differentiable at c, then prove that f is continuous at c. (A, CO 2)
13. Examine the function $x^3 + y^3 - 3axy$ for maxima and minima. (An, CO 3)
14. Verify that the mixed partial derivatives $D_{1,2}\mathbf{f}$ and $D_{2,1}\mathbf{f}$ are equal where $\mathbf{f}(x, y) = \tan(x^2/y), \quad y \neq 0$. (A, CO 3)
15. Find $J_f(r, \theta, z)$ where $f(r, \theta, z)$ is defined by $x = rcos\theta, y = rsin\theta, z = z$ (An, CO 4)
16. Find the gradient vector $\nabla f(x, y, z)$ at the point $(1, 0, 1)$ of the function $f(x, y, z) = 3x^3 + y^4 + z^5$. (A, CO 2)
17. Derive the exponential form of the Fourier Integral Theorem. (A, CO 1)
18. Prove that $\frac{x^2}{2} = \pi x - \frac{\pi^2}{3} + 2\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ if $0 \leq x \leq 2\pi$. (A, CO 1)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. State and prove Fourier Integral theorem. (A, CO 1)

20. Assume that one of the partial derivatives $D_1\mathbf{f}, \ldots, D_n\mathbf{f}$ exists at $\mathbf{c}$ and that the remaining $n-1$ partial derivatives exists in some open ball and are continuous at $\mathbf{c}$. Then show that $\mathbf{f}$ is differentiable at $\mathbf{c}$. (An, CO 3)

21. Suppose E is an open set in R^n, T is a C'-mapping of E into an open set $V \subset R^m$. Let ω and λ be k- and m- forms in V respectively. Then prove that
(a) $(\omega + \lambda)_T = \omega_T + \lambda_T$ if $k = m$;
(b) $(\omega \wedge \lambda)_T = \omega_T \wedge \lambda_T$;
(c) $d(\omega_T) = (d\omega)_T$ if ω is of class C' and T is of class C''. (An, CO 4)

22. Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b, with total derivative $f'(b)$. Then prove that the composite function $h = f \circ g$ is differentiable at a, and the total derivative $h'(a)$ is given by $h'(a) = f'(b) \circ g'(a)$, the composition of the linear functions $f'(b)$ and $g'(a)$. (A, CO 2)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain Weirstras theorem, otherforms of Fourierseries, the Fourier integral theorem, the exponential form of the Fourier integral theorem, integral transforms and convolutions, the convolution theorem for Fourier transforms.	A	1, 3, 7, 17, 18, 19	12
CO 2	Analyze Multivariable Differential Calculus The directional derivative, directional derivatives and continuity, the total derivative, the total derivative expressed in terms of partial derivatives, An application of the complex- valued functions, the matrix of a linear function, the Jacobian matrix, the chain rate matrix form of the chain rule.	A	2, 9, 12, 16, 22	11
CO 3	Interpret Implicit functions and extremum problems, the mean value theorem for differentiable functions, a a sufficient condition for differentiability.	An	6, 10, 13, 14, 20	11
CO 4	Explain the Integration of Differential Forms, primitive mappings, partitions of unity, change of variables, differential forms, and Stoke's theorem.	An	4, 5, 8, 11, 15, 21	12

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;