Reg. No .... Name **23P352**

M.Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT14 : ADVANCED COMPLEX ANALYSIS**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. State and prove the functional equation. (A)
2. State Mittag-Leffler's theorem. (R)
3. Show that an analytic function whose real part is constant must itself be a constant. (A, CO 3)
4. Let G be an open connected subset of (C). Then tshow that if G is simply connected, then $n(\gamma; a) = 0$ for every closed rectifiable curve γ in G and every point a in $C - G$ (U, CO 2)
5. State Hadamard factorization theorem. (E)
6. State Reimann mapping theorem. (A)
7. State Jensen's Formula. (R, CO 4)
8. FInd the order of the functions $a)\sin(z) \quad b)cosh\sqrt{z}$ (A, CO 4)
9. Let G be an open connected subset of C. For any f in $H(G)$ there is a sequence of polynomials that converges to f in $H(G)$ then show that for any f in $H(G)$ and any closed rectifiable curve γ in G, $\int_\gamma f = 0$ (A, CO 2)
10. Show that if u is harmonic then show that so are u_x and u_y (An, CO 3)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. State and prove Maximum Principle (secondVersion). (U, CO 3)
12. Show that C and $D = \{z / |z| \leq 1\}$ are homeomorphic. (A, CO 2)
13. Let $Rez_n \leq -1$; then the series $\sum_{n=1}^{\infty} log(1 + z_n)$ converges absolutely iff the series $\sum_{n=1}^{\infty} z_n$ converges absolutely. (E)
14. Let G be an open connected subset of C. If for any f in $H(G)$ such that $f(z) \neq 0$ for all z in G there is a function g in $H(G)$ such that $f(z) = [g(z)]^2$, then show that If $u : G \longrightarrow R$ is harmonic then there is a harmonic function $v : G \longrightarrow R$ such that $f = u + iv$ is analytic on G (U)
15. Let f be an analytic function on a region containing $\bar{B}(0, r)$ and suppose that $a_1, \ldots, a_n$ are the zeros of f in $B(0; r)$ repeated according to multiplicity. If $f(0) \neq 0$ then $\log|f(0)| = -\sum_{k=1}^{n} \log(\frac{r}{|a_k|}) + \frac{1}{2\pi}\int_0^{2\pi} \log|f(re^{i\theta}|\, d\theta$ (R, CO 4)

16. State and prove Bohr Mollerup Theorem. (U)

17. State and prove Euler'stheorem. (U)

18. If $\mathbf{f}(\mathbf{z}) = \boldsymbol{u} + \boldsymbol{iv}$ is a regular function of z in a domain D, then show that $\boldsymbol{\nabla}^{\mathbf{2}}|\boldsymbol{f}(\boldsymbol{z})|^{\mathbf{2}} = \mathbf{4}|\boldsymbol{f}^{'}(\boldsymbol{z})|^{2}$ (An, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Show that f is analytic on D iff $\int_{-\pi}^{\pi} f(e^{it})e^{int}dt = 0$ for all $n \geq 1$. (A)

20. State and prove Runge's theorem. (An, CO 2)

21. Let $\prod_{n=1}^{\infty} X_n, d)$, is a metric space. If $\xi^k = \{x_n^k\}_{n=1}^{\infty}$ is in $\prod_{n=1}^{\infty} X_n$, then prove that $\xi^k \longrightarrow \xi = \{x_n\}$ iff $x_n^k \longrightarrow x_n$ for each n. Also show that if each (X_n, d_n) is compact then X is compact. (R)

22. Show that $H(G)$ is a complete metric space. (A)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain the space of functions, Riemann mapping theorem and Weierstrass factorization theorem.	U	22	5
CO 2	Analyze Runge's Theorem, Simple connectedness, MittagLeffler's theorem, Analytic continuation and Riemann surfaces, Schwartz Reflection Principle, Analytic continuation along a path, Mondromy theorem.	An	4, 9, 12, 20	9
CO 3	Interpret Harmonic functions, Basic properties of harmonic functions and Harmonic functions on the disk.	U	3, 10, 11, 18	6
CO 4	Perceive Entire functions, Jensen's formula, the genus and order of an entire function, Hadamard Factorization theorem.	U	7, 8, 15	4

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;