Reg. No .... Name **23P337**

M Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT13 : ADVANCED TOPOLOGY**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Let $X^+ = XU\{\infty\}$ be the one point compactification of the space X. If X is compact, prove that $\{\infty\}$ is open in X^+. ()
2. Prove that T_2 axiom is a productive property. ()
3. Define base and sub base of a filter on a set X. (Cr)
4. If a topological space is embeddedable into a cube, prove that the space is Tychonoff. ()
5. If every net in a space X can converge to any point in it, then prove that X is indiscrete. ()
6. State Urysohn's Lemma. (An)
7. Define σ- locally finite family in a topological space. ()
8. If a space X is Hausdorff, prove that no filter on X can converge to more than one point in it. ()
9. Is $\mathbb{R}$ locally compact? Justify. ()
10. Define a filter associated with a net S in X. ()

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Prove that a topological product is regular iff each co-ordinate space is regular. ()
12. Prove that a space X is connected if and only if each co-ordinate space is so.. (U)
13. Prove that a first countable, countably compact space is sequentially compact. ()
14. Give an example of a space which is second countable and T_2 but not metrisable. ()
15. Prove that a space X is compact iff every filter on X has a cluster point in X. ()
16. Prove that open subspace of a locally compact, regular space is locally compact. ()
17. Prove that the evaluation function is 1 - 1 if and only if f_i distinguishes points and that the evaluation function is continuous iff each f_i is continuous. (R)
18. Define co-final subset of a directed set. Suppose $S : D \to X$ is a net and F is a co-final subset of S. If $S/F : F \to X$ converges to a point x in X, then prove that x is a cluster point of S. ()

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let $S : D \to X$ be a net and $\mathcal{F}$ the filter associated with it. Let $x \in X$. Prove that ()

    a. S converge to x as a net iff $\mathcal{F}$ converges to x as a filter.
    b. x is a cluster point of the net S iff x is a cluster point of the filter $\mathcal{F}$

20. If the product space is non-empty, prove that each co-ordinate space is embeddable in it and hence prove that if a topological product is T_0, T_1, T_2 or regular, then each co-ordinate space has the corresponding property. ()

21. Prove that a space second countable T_3 space if and only if it is embeddable in a Hilbert cube. (U)

22. Prove that a subspace of a locally compact, Hausdorff space is locally compact iff it is open in its closure. ()

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;