Reg. No .... Name **23P322**

M Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT12 : FUNCTIONAL ANALYSIS**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. If a nls X has a Schauder basis, show that X is separable. (E, CO 2)
2. Let X be a Banach space and $A \in BL(X)$. Prove that the map defined on the set of all invertible elements in $BL(X)$, which takes an invertible element $A \in BL(X)$ to its inverse A^{-1} in $BL(X)$, is continuous. (U, CO 3)
3. Let X and Y be normed linear spaces of dimension n. Show that X and Y are linearly homeomorphic. (A, CO 1)
4. Define finite rank operator. Also define rank of an operator. (U, CO 3)
5. Explain the concept of normed linear space. Give an example. (U, CO 1)
6. Define the dual space of a normed linear space X. (R, CO 4)
7. Define spectral value of an operator $A \in BL(X)$. Determine the spectral values of the identity operator in $BL(X)$. (U, CO 3)
8. Define a convex subset of a linear space X. Give an example of a convex subset in $\mathbb{R}^2$. (U, CO 1)
9. When are two normed linear spaces said to be linearly homeomorphic? When are two normed linear spaces said to be linearly isometric? (U, CO 1)
10. State the Banach - Steinhaus theorem. (U, CO 2)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Let X be a Banach space. Suppose Y and Z are two closed subspaces of X with $Y + Z = X$ and $Y \cap Z = \{0\}$. Let $x = y + z$, where $y \in Y$ and $z \in Z$. Show that the map $P : X \to X$ defined by $P(x) = y$ is continuous. (An, CO 2)
12. Construct a discontinuous linear functional on any infinite dimensional nls X. (An, CO 2)
13. If an operator A is of finite rank on a linear space, show that $A - I$ is one-one if and only if $A - I$ is onto. (An, CO 3)
14. Let X be a normed linear space. Show there exists a unique Banach space X_c and a linear isometry F_c of X into X_c such that $F_c(X)$ is dense in X_c. (An, CO 4)
15. State and prove Riesz Lemma. (An, CO 1)
16. Let X and Y be normed linear spaces and $F : X \to Y$ be a linear map. Show that F is continuous at zero if and only if $||F(x)|| \leq \alpha||x||$ for all $x \in X$ and some $\alpha > 0$. (E, CO 1)

17. Let X be a normed linear space. Prove that if X is finite dimensional, the closed unit ball in X is compact. (An, CO 1)

18. Does a convergent sequence of finite rank operators on $BL(X)$, where X is a Banach space, always have a finite rank operator as its limit? Justify your answer. (An, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let X and Y be Banach spaces and $F \in BL(X, Y)$. Prove that F is onto if and only if F' is bounded below. (E, CO 4)

20. State and prove the Hahn-Banach extension theorem. (E, CO 1)

21. Let X be a normed linear space and Y be a closed subspace of X. Show that Y with induced norm and X/Y with the quotient norm are Banach if and only if X is Banach. (An, CO 2)

22. Let X be a Banach space over $\mathbb{C}$ and $A \in BL(X)$. Prove that $s(A) \neq \emptyset$ (An, CO 3)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Analyze normed linear spaces, continuity of linear maps , theory and applications of the Hahn-Banach Theorems	E	3, 5, 8, 9, 15, 16, 17, 20	15
CO 2	Analyze Banach spaces, Uniform boundedness principle and the Closed graph theorem.	E	1, 10, 11, 12, 21	11
CO 3	Analyze the Open Mapping theorem, the eigen spectrum and spectral radius.	E	2, 4, 7, 13, 18, 22	12
CO 4	Analyze duals of a normed linear space and transposes of bounded linear maps.	E	6, 14, 19	8

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;