Reg. No .... Name **23U546**

B. Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 5 : COMPUTER APPLICATION

COURSE : **19U5CRCMT6 : MATHEMATICAL ANALYSIS**

(For Regular 2021 Admission and Supplementary 2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Show that the set $S = \{x/0 < x < 1, x \in R\}$is open but not closed.
2. Find the supremum and infimum of $\left\{1 + \frac{(-1)^n}{n};\ n \in \mathbb{N}\right\}$.
3. (a) Define upperbound of a set S
   (b) Find the supremum of the set $\left\{\frac{1}{\mathrm{n}} : \mathrm{n} \in \mathbb{N}\right\}$
4. Sketch the set of points determined by $|z - 1 + i| = 1$.
5. a) Define deleted neighbourhood of a point with an example.
   b) Give an example of a set which is neither open nor closed.
6. Verify that (2-3i)(-2+i)=-1+8i.
7. Show that if a set S is bounded then so is its closure $\tilde{s}$.
8. State the equivalence of two forms of Completeness property of real numbers.
9. Show that the sequence $\{S_n\}$, where S_n=$1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{n}$ cannot converge.
10. Represent the complex number $z = 1 + i\sqrt{3}$ in exponential form.
11. Show that $\lim\limits_{n \to \infty} \frac{3+2\sqrt{n+1}}{\sqrt{n+1}} = 2$.
12. a) State Cauchy's first theorem on limits.
    b) Give an example to show that converse of Cauchy's first theorem on limits is not true.

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. If a sequence $\{a_n\}$ of positive terms converges to a positive limit l,then so does the sequences $\left\{(a_1 a_2 a_3 \ldots\ldots\ldots a_n)^{1/n}\right\}$ of its geometric means.
14. Find $(1)^{1/3}$.
15. Show that if **R** is order complete, then it has Dedekind's property.
16. (a) Reduce $\frac{1+2\mathrm{i}}{3-4\mathrm{i}} + \frac{2-i}{5i}$ into real number
    (b) Write $-1 - \mathrm{i}$ in polar form
17. Prove that the derived set of a set is closed.
18. Prove that every bounded infinite set has the smallest and greatest limit points.

19. Let A,B ⊆ **R** such that A ⊆ B. Show that Sup A ≤ Sup B

20. Show thtat the sequence {S_n},
where $S_n = \frac{1}{1!} + \frac{1}{2!} + \ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{n!},\ \forall n \in \mathbb{N}\ is\ convergent.$

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. (a) Show that the real number field is Archimedean.
(b) Prove that every open interval (a,b) contains a rational number.

22. a) If {a_n} be a sequence, such that $lim \frac{a_{n+1}}{a_n} = l,\ where\ |l| < 1,\ then\ prove\ that\ lim\ a_n = 0.$
b) Prove that a monotonic sequence is convergent if and only if it is bounded.

23. a) Prove that every bounded infinite set has the smallest and greatest limit points
b) Prove that closure of a bounded set is bounded

24. Show that the sequences$\{{a_n}^{1/n}\} and\ \left\{{b_n}^{1/n}\right\}\ where\ \left(i\right)\ a_n = \frac{(3n)!}{(n!)^3}\ and\ \left(ii\right) b_n = \frac{n^n}{(n+1)(n+2)\ldots\ldots(n+n)}$
converge and hence find their limits.

(10 x 3 = 30)