Reg. No .... Name **23U540**

B. Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 5 : MATHEMATICS

COURSE : **19U5CRMAT7 : ALGEBRA**

(For Regular 2021 Admission and Supplementary 2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Show that if R is a ring, then for any $a, b \in R,\ a(-b) = (-a)b = -(ab)$.
2. Define commutative binary operaton.
3. Define projection map on the direct product of groups.
4. Define proper subgroup of a group.
5. Define a conjugate subgroup of a subgroup.
6. Define simple group.
7. Define permutation on a set.
8. Find the number of elements in the set $\{\sigma \in S_4 | \sigma(3) = 3\}$.
9. True or false: every function is permutation if and only if it is one to one. Justify.
10. Define order of an element.
11. Show that there is no zero divisor in $\mathbb{Z}_p$, where p is a prime.
12. Define unit element of a ring.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Prove that direct product of the groups is a group.
14. Let m be a positive integer and let $a \in \mathbb{Z}_m$ be relatively prime to m. Show that for each $b \in \mathbb{Z}_m$, the equation $ax = b$ has a unique solution in $\mathbb{Z}_m$.
15. Show that $(\mathbb{Z}_4 \times \mathbb{Z}_2)/(\{0\} \times \mathbb{Z}_2) \cong \mathbb{Z}_4$.
16. Prove that every cyclic group is abelian.
17. Let R be a ring with unity. Show that if $n.1 = 0$ for some natural number n, then the smallest such integer n is the characteristic of R.
18. Prove that the group homomorphism $\phi : G \to G'$ is a one to one map if and only if Ker$(\phi) = \{e\}$.
19. Show that $*$ defined on $\mathbb{Q}^+$ by $a * b = ab/2$ makes $\mathbb{Q}^+$ an abelian group.
20. Show that the group $\prod_{i=1}^{n} \mathbb{Z}_{m_i}$ is cyclic if and only if the numbers m_i are relatively primes in pairs.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Compute the maximum possible order of an element in A_{10}.
22. Let S be any subset of a group G. Show that $H_S = \{x \in G | xs = sx, \forall s \in S\}$ is a subgroup of G. Hence show that the center of G is an abelian group.
23. State and prove fundamental homomorphism theorem.
24. Let R be a ring with $a^2 = a, \ \forall a \in R$. Prove that R is a commutative ring.

(10 x 3 = 30)