

B.Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023**SEMESTER 5 : MATHEMATICS****COURSE : 19U5CRMAT06 : DIFFERENTIAL EQUATIONS***(For Regular 2021 Admission and Supplementary 2020 / 2019 Admissions)*

Time : Three Hours

Max. Marks: 75

PART A**Answer any 10 (2 marks each)**

1. Find the regular singular point of $x^2(x-2)^2 \frac{d^2y}{dx^2} + 2(x-2) \frac{dy}{dx} + (x+1)y = 0$?
2. Define Bernoulli's equation.
3. Check whether e^x is a solution of $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{y}{x} + 2y = 0$
4. If the ratio of the indicial equations r_1 and r_2 are such that $r_1 - r_2 = N$, where N is a positive integer, then write the two linearly independent solutions.
5. Find the general solution of $\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} = 0$
6. Form a partial differential equation from the equation $z = (x^2 + a^2)(y^2 + b^2)$ by eliminating arbitrary constants.
7. What is the regular singular point of $2x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + (x-5)y = 0$?
8. What is the unique solution of $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + 4x\frac{y}{x} + x^2y = 0$ such that $f(2) = f'(2) = f''(2) = 0$
9. Define orthogonal trajectory.
10. Form a partial differential equation by eliminating arbitrary constants from $z = ax + by + a^2 + b^2$
11. What is the transformation that is used to solve Bernoulli's equation.
12. What is the auxiliary equation of the Lagrange's linear partial differential equation?
(2 x 10 = 20)

PART B**Answer any 5 (5 marks each)**

13. Given that $y = x$ is a solution of $x^2y'' - 4xy' + 4y = 0$. Find a linearly independent solution and write the general solution
14. Find the general solution of the differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z$
15. Solve $\frac{dx}{y^2 + z^2} = \frac{dy}{-xy} = \frac{dz}{-xz}$
16. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + 4e^{3x}$.
17. Use the operator method to solve the following system of equations
 $\frac{dx}{dt} + \frac{dy}{dt} - 2x - 2y = e^t, \frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$
18. Find the orthogonal trajectories of $y = cx^3$
19. Solve $(3y + 2x + 4)dx - (4x + 6y + 5)dy = 0$.
20. Find the general solution of $y^{11} + y = 0$ in terms of power series in X .

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. (a) Find the integral curves of the equation $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$
(b) Find the general solution of the differential equation $x^2p + y^2q = (x+y)z$
22. Solve $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$.
23. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x}$.
24. Use the method of Frobenius to find the solution of $2x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + (x-5)y = 0$ in some interval $0 < x < R$

(10 x 3 = 30)