Reg. No ..... Name **23P307**

MSc.DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT11 : PARTIAL DIFFERENTIAL EQUATIONS**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Find the complete integral of $p + q = pq$. (A, CO 2)
2. Show that the pdes $z = px_1 + qx_2$ and $f(x_1, x_2, z, p, q) = 0$ are compatible if $f(x_1, x_2, z, p, q) = 0$ is homogeneous in x_1, x_2, z. (A, CO 2)
3. Form pde by eliminating arbitrary function $z = x_1 + x_2 + F(x_1 x_2)$. (A, CO 1)
4. State Dirichlet problem for a rectangle. (R, CO 4)
5. Solve $(D^3 - 2D^2D' - DD'^2 + 2D'^3)z = 0$. (A, CO 3)
6. Solve $(2D - D' + 4)(D + 2D' + 1)z = 0$. (A, CO 3)
7. Find the complete integral of $(p + q)(z - px_1 - qx_2) = 1$. (A, CO 2)
8. Classify the pde as elliptic, hyperbolic or parabolic $z_{xx} + z_{yy} = 0$. (U, CO 4)
9. Find the first order pde satisfied by the homogeneous function $f(x_1, x_2)$ of x_1 and x_2 of degree n. (A, CO 1)
10. Let $v = v(x_1, x_2)$ be a known function of x_1, x_2 and $u = H(v)$ be a function of v alone not involving x_1, x_2 explicitly and H has continuous first order derivatives. Find the first order pde. (An, CO 1)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Show that the equation $(x_2x_3)dx_1 + (2x_1x_3)dx_2 + (-3x_1x_2)dx_3 = 0$ is integrable and find the integral. (A, CO 1)
12. Let $\alpha_r D + \beta_r D' + \gamma_r$ is a factor of $F(D, D')$ and $\phi_r(\xi)$ is an arbitrary function of the variable ξ. Prove that if $\alpha_r \neq 0, u_r = \exp(\frac{-\gamma_r x}{\alpha_r})\phi_r(\beta_r x - \alpha_r y)$ is a solution of the equation $F(D, D')z = 0$. (A, CO 3)
13. Show that the equations $p^2 + q^2 - 1 = 0, (p^2 + q^2)x_1 - pz = 0$ are compatible and find a one parameter family of common solutions. (An, CO 2)
14. Let $z = F(x_1, x_2, a)$ be a one parameter family of solutions of the pde $f(x_1, x_2, z, p, q) = 0$.Then prove that the envelope of this one parameter family if it exists is also a solution of the pde. (A, CO 1)
15. Derive Charpit's method for solving non linear pde. (A, CO 2)
16. Solve: $(r + s - 2t)z = e^{2x+y}$ (A, CO 3)
17. Solve $q^2r - 2pqs + p^2t = 0$ using Monge's method. (A, CO 4)
18. By seperating the variables , show that the equation $z_{xx} = \frac{1}{k}z_t$ has solution of the form $z(x, t) = \sum_0^\infty c_n cos(nx + \epsilon_n)e^{-n^2kt}$. (A, CO 4)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Solve using Monge's method $r + (a + b)s + abt = xy$. (An, CO 4)

20. (i) Solve $(D^2 - D')z = 2y - x^2$
(ii) Solve $(D^2 - D')z = e^{2x+y}$ (A, CO 3)

21. Derive the condition for compatibility of first order pde's. (An, CO 2)

22. Prove that a necessary and sufficient condition for a Pfaffian differential equation $X.dr = 0$ to be integrable is that $X.curlX = 0$. (An, CO 1)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	explain the classification of first order pde and their solutions	A	3, 9, 10, 11, 14, 22	12
CO 2	illustrate the integrals of nonlinear pde's	An	1, 2, 7, 13, 15, 21	12
CO 3	analyze linear pde with constant coefficients and special second order pde's	An	5, 6, 12, 16, 20	11
CO 4	analyze solutions of Laplace's equation	An	4, 8, 17, 18, 19	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;