Reg. No .................... Name .............................. **23U504**

**B. Sc. DEGREE END SEMESTER EXAMINATION : NOVEMBER 2023**

**SEMESTER 5 : MATHEMATICS**

COURSE : **19U5CRMAT05 : REAL ANALYSIS - I**

*(For Regular 2021 Admission and Supplementary 2020 / 2019 Admissions)*

Time : Three Hours Max. Marks: 75

**PART A**

**Answer any 10 (2 marks each)**

1. State the Comparison Test ( First Type) for series.
2. Show that $\lim_{x \to 3} \dfrac{1}{(x-3)^4} = \infty$
3. Give an example each of (a) a convergent sequence (b) a divergent sequence.
4. Show that a finite set has no limit points.
5. Show that the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$ is not convergent.
6. State Cauchy's second theorem on limits.
7. Define countably infinite set. Give an example.
8. State the Leibnitz test for convergence of alternating series.
9. Define limit point of a sequence. What are the limit points of the sequence $\{(-1)^n(1+\frac{1}{n})\}$?
10. Give an example of a subset of $\mathbb{R}$ which is not order complete.
11. Prove that $|x| = max(x, -x)$.
12. State Cauchy's first theorem on limits.

**(2 x 10 = 20)**

**PART B**

**Answer any 5 (5 marks each)**

13. Show that $\lim_{x \to 4} f(x) = \begin{cases} \dfrac{|x-4|}{x-4} & x \neq 4 \\ 0 & x = 4 \end{cases}$ does not exist.
14. Show that Cauchy's root test fails to give any information about the convergence of a series $\sum u_n$ when $\lim_{n \to \infty} (u_n)^{1/n} = 1$.
15. Show that $\lim_{n \to \infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.
16. Prove that a monotonic increasing sequence which is bounded above converges to its supremum.
17. If $S$ and $T$ are subsets of real numbers, prove that $(S \cup T)' = S' \cup T'$.
18. Show that if $p > 1$, the positive term series $\sum \dfrac{1}{n^p}$ converges.
19. Show that if $M$ and $N$ are neighbourhoods of a point $x$, then $M \cap N$ is also a neighbourhood of $x$.

20. Discuss the convergence of the series $\sum \frac{n}{n^2+1} x^n$, where $x > 0$

**(5 x 5 = 25)**

**PART C**

**Answer any 3 (10 marks each)**

21. State Raabe's test for convergence. Using it test for convergence the series $1 + \frac{\alpha}{\beta}x + \frac{\alpha(\alpha+1)}{\beta(\beta+1)}x^2 + \frac{\alpha(\alpha+1)(\alpha+2)}{\beta(\beta+1)(\beta+2)}x^3 + \cdots$

22. State and prove Cauchy's general principle of convergence.

23. State the logarithmic test for convergence of a positive term series . Discuss the convergence of the series $1 + \frac{x}{1!} + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \ldots$ for $x > 0$.

24. (a) Show that the set of real numbers in $[0, 1]$ is uncountable.
(b) Show that the set of rational numbers in $[0, 1]$ is countable.

**(10 x 3 = 30)**