Reg. No ....   Name **23U457**

B. Sc. DEGREE END SEMESTER EXAMINATION : MARCH 2023
SEMESTER 4 : COMPLEMENTARY MATHEMATICS FOR PHYSICS AND CHEMISTRY
COURSE : **19U4CPMAT04 : Fourier Series,Laplace Transforms, Fourier Transforms, and Groups.**
(For Regular - 2021 Admission and Improvement / Supplementary - 2020 / 2019 Admissions)

Time : Three Hours   Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Using Fourier sine integral, show that
$$\int_0^\infty \frac{1-\cos \pi\lambda}{\lambda}\sin x\lambda\, d\lambda = \begin{cases} \frac{\pi}{2} & when \quad 0 < x < \pi \\ 0 & when \quad x > \pi \end{cases}$$

2. State the Dirchlet's conditions

3. Find the Laplace transform of $e^{at}\cos at$

4. Define Laplace transform and find the Laplace transform of t^n, n is an integer

5. Define the Fourier Cosine Transform.

6. Find the Laplace transform of $\cos hat$

7. Define the Fourier cosine integral of $f(x)$.

8. Is the binary operation on $\mathbb{Z}$ defined by $a*b = \frac{ab}{2}$ associative? Justify.

9. Define periodic functions. Check weather the function $sinax$ is periodic

10. Is the binary operation on $\mathbb{Z}$ defined by $a*b = a-b$ commutative? Justify.

11. Define a cyclic group. Give an example

12. Find the Fourier coefficient a_0 for the function $f\left(x\right) = \frac{1}{4}(\pi-x)^2,\ 0 < x < 2\pi$

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. Explain the group D_4. Write down its elements and its subgroups.

14. Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$

15. Find the Fourier sine transform of $e^{-|x|}$. Hence evaluate $\int_0^\infty \frac{x\sin mx}{1+x^2}\, dx$.

16. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$

17. Expand $f(x) = e^{-x}$ as a Fourier series in the interval $(-l, l)$

18. Show that the identity element in a group is unique. Further show that the inverse of each element in a group is unique.

19. Find the inverse Laplace transform of $\frac{5s+3}{(s+1)(s^2+2s+5)}$

20. Express the function $f(x) = \begin{cases} 1 & for \quad |x| < 1 \\ 0 & for \quad |x| > 1 \end{cases}$ as a Fourier integral.Hence evaluate $\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}\, d\lambda$

(5 x 5 = 25)

21. Find the half range Fourier sine and cosine series of the

    function $f\left(x\right) = \begin{cases} x & 0 < x < \frac{\pi}{2} \\ \pi - \text{x} & \frac{\pi}{2} < x < \pi \end{cases}$

22. Solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = 3\cos 3t - 11\sin 3t, \;\; y\left(0\right) = 0 \; and \; y'\left(0\right) = 6$

23. Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$

24. (a) Draw the group tables of $\mathbb{Z}_4$ and the Klein 4 group V.
    (b) Show that $\mathbb{Z}_4$ is cyclic and the Klein 4 group V is not cyclic.
    (c) Write down the subgroups of Z_4 and the subgroups of the Klein 4 group V.

    (10 x 3 = 30)