$\qquad$ Name $\qquad$

# MSc DEGREE END SEMESTER EXAMINATION- NOVEMBER 2017 

SEMESTER 1 : PHYSICS

## COURSE : 16P1PHYTO2 ; CLASSICAL MECHANICS <br> (For Regular - 2017 Admission)

Time : Three Hours
Max. Marks: 75

## Section A (Objective type) <br> Answer all the questions (1 Mark each)

1. The number of degrees of freedom of a rigid body is
(a) 2
(b) 6
(c) 9
(d) 3
2. Which of the following equation does not represent Hamilton's principle for a conservative system
(a) $\delta \int p d q=0$
(b) $\delta \int T d t=0$
(c) $\delta H>0$
(d) $\delta S=0$
3. If the generating function has the form $F=F\left(q_{j}, P_{j}, t\right)$
(a) $p_{j}=\frac{\partial F}{\partial q_{j}}, Q_{j}=\frac{\partial F}{\partial P_{j}}$
(b) $p_{j}=\frac{-\partial F}{\partial q_{j}}, Q_{j}=\frac{\partial F}{\partial P_{j}}$
(c) $p_{j}=\frac{\partial F}{\partial q_{j}}, Q_{j}=\frac{-\partial F}{\partial P_{j}}$
(d) $p_{j}=\frac{-\partial F}{\partial q_{j}}, Q_{j}=\frac{-\partial F}{\partial P_{j}}$
4. If a rigid body is rotating with an angular velocity ' $w$ ' about an instantaneous axis through a fixed point in the body, the angular momentum vector $\vec{J}$ about the same point
(a) will be always in the direction of $w$
(b) can be in the direction of $w$
(c) will always perpendicular to ' $w$ '
(d) will never be in the direction of $w$.
5. Which of the following statement is true about chaotic systems
(a) chaotic systems can either be dissipative or conservative
(b) In dissipative systems phase-space volumes contract
(c) In conservative system phase space volumes are conserved
(d) All of these

## Section B (Short answer type) <br> Answer any Seven (2 marks each)

6. Prove that the system for which the KE is conserved, moves along that path for which the time of transit is extremum.
7. Differentiate between conservative and dissipative systems.
8. For a free particle $H=T$ and $L=T$. Hence from Hamilton's equations $\dot{p}=\frac{-\partial T}{\partial q}$ and from Lagrange's equations $\dot{p}=\frac{\partial T}{\partial q}$. How do you reconcile the two equations?
9. Obtain poisson bracket $\left[L_{x}, L_{y}\right]$, where $L_{x}$ and $L_{y}$ are $x$ and $y$ components of angular momentum.
10. Explain how the method of action angle variables provides a procedure for quantization of systems.
11. What are principal axes and principal moment of inertia of a rigid body?
12. Show that a non-inertial frame is violating Newton's second law of motion.
13. If the rotation axis of a body is in the direction of principal axis, show that the angular velocity vector and angular momentum will be in the same direction.
14. What are the characteristics of a strange attractor?
15. Differentiate between chaotic system and an attractor.

## Section C (Problems / Short Essays) <br> Answer any Four (5 marks each)

16. In a spherical pendulum the bob of mass ' $m$ ' is constrained to move on a spherical surface of radius $R ; R$ being the length of the pendulum. Set up the Lagrangian for the spherical pendulum and obtain the equations of motion.
17. Three masses $m_{1}, m_{2}$ and $m_{3}$ are attached with a spring with $m_{2}$ in the middle and with $m_{1}=m_{3}$. Obtain the modes of vibration of this system.
18. Two identical simple pendulums, each of length ' $l$ ', are connected by a light spring of force constant ' $k$ '. If ' $m$ ' is the mass of each bob, show that the normal frequencies of the system are, $w_{1}=\sqrt{\frac{g}{l}}$ and $w_{2}=\sqrt{\frac{g}{l}+\frac{2 k}{m}}$.
19. A rigid body is rotating under the influence of an external torque ' $N$ ' acting on it. If ' $w$ ' is the angular velocity and $T$ is the kinetic energy, show that $\frac{d T}{d t}=N . w$, in the principal axes system.
20. Choosing the origin at any one corner, obtain the inertia tensor of a rectangular parallelopiped of density ' $\rho$ ' and sides $a, b, c$. Hence deduce the inertia tensor for a cube of side ' $a$ '.
21. Show how the iterates of the 2D-Baker's map form a Cantor set pattern.

## Section D (Essays)

Answer all questions ( $\mathbf{1 2}$ marks each)
22(a) Discuss calculus of variations and derive Lagrange's equations from Hamilton's principle.
(b) Discuss calculus of variation. Show that the integeral $I=\int_{x_{1}}^{x_{2}} f\left(y, y^{\prime}, x\right) d x$ is stationary, when $\frac{d}{d x}\left(\frac{\partial f}{\partial y^{\prime}}\right)-\frac{\partial f}{\partial y}=0$, where $y^{\prime}=\frac{d y}{d x}$.
23(a) Use the Hamilton-Jacobi method to determine the motion of a particle falling vertically in a unfiorm gravitational field.

OR
(b) Obtain the equations of motion and first integrals of a particle moving in a central force field.
24(a) Explain the rate of change of a vector and derive an expression for the Coriolis force.

OR
(b) Obtain the pendulum equation by considering it as a non-linear system. Obtain the phase portrait of the pendulum also.

