PART A
Answer any 10 (2 marks each)

1. Find the general solution of $\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 29y = 0$.

2. Find the general solution of $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 12\frac{dy}{dx} - 8y = 0$.

3. Locate and classify the singular points of the differential equation $\left(x^2 - 3x\right)\frac{d^2y}{dx^2} + (x+2)\frac{dy}{dx} + y = 0$.

4. Find the general solution of $\frac{d^5y}{dx^5} - 3\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 0$.

5. Find the integral curves of the equations $\frac{dx}{1} = \frac{dy}{-1} = \frac{dz}{1}$.

6. Examine that the differential equation $x^{-2}y\,dx + \left(y - x^{-1}\right)dy = 0$ is an exact differential equation.

7. Find the singular points of $\left(x^3 + x^2\right)y'' + \left(x^2 - 2x\right)y' + 4y = 0$.

8. Find the general solution of $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$.

9. Form the partial differential equation by eliminating the constants a and b from $z = ax + by + \sqrt{a^2 + b^2}$.

10. Find the general integral of the linear partial differential equation $p\,\tan x + q\,\tan y = \tan z$.

11. Examine that the equation $\left(y^2 + yx\right)dx + x^2\,dy = 0$ *is homogeneous.*

12. Solve the linear differential equation $\frac{dy}{dx} - y = e^{2x}$.

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. Find the integral curves of the equations $\frac{dx}{x^2} = \frac{dx}{y^2} = \frac{dz}{(x+y)z}$.

14. Given that $y=x^{-1}$ is a solution of $2x^2y'' + 3xy' - y = 0$. (x>0), find a linearly independent solution by reducing the order.

15. Solve the initial value problem $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 100\frac{dy}{dx} - 100y = 0, y\left(0\right) = 4, y'\left(0\right) = 11, y''\left(0\right) = -299$.

16. Solve the differential equation $\left(3x^2 + 4xy\right)dx + \left(2x^2 + 2y\right)dy = 0$.

17. Solve $2r\left(s^2 + 1\right)dr + \left(r^4 + 1\right)ds = 0$.

18. Find the integral curves of the equations $\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)}$.

19. Solve the initial value problem $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$, $y\left(0\right) = 1$, $y'\left(0\right) = 6$.

20. Prove that $\frac{d}{dx}\left(x\,J_1\left(x\right)\right) = x\,J_0\left(x\right)$.

(5 x 5 = 25)

PART C
Answer any 3 (10 marks each)

21. Define an orthogonal trajectory. Find the orthogonal trajectory of the family of circles $\left(x - c\right)^2 + y^2 = c^2$.

22. i) Find the general solution of the differential equation $x^2p + y^2q = (x + y)z$.
    ii) Form the PDE by eliminating the arbitary function from the equation $z = f(2x + y) + g(3x - y)$.

23. Solve the differential equation $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^x$ by the method of variation of parameters.

24. Solve the system of equations using operator method:
    $\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t$,
    $\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$

(10 x 3 = 30)