Reg. No ..... Name **23U426**

B. Sc. DEGREE END SEMESTER EXAMINATION : MARCH 2023

SEMESTER 4 : MATHEMATICS

COURSE : **19U4CRMAT4 : ANALYTIC GEOMETRY NUMERICAL METHODS AND NUMBER THEORY**

(For Regular - 2021 Admission and Improvement / Supplementary - 2020 / 2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Find the nature of the conic $\frac{8}{r} = 4 - 5\cos\theta$.
2. Use bisection method to find the real root of the equation $f(x) = x^3 - x - 1 = 0$.
3. Find the equation of director circle to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
4. Find the points on the conic $\frac{9}{r} = 2 + \sqrt{2}\cos\theta$ whose radius vector is 3.
5. Show that if $a \equiv b(\mod n)$, then $a^k \equiv b^k(\mod n)$ for any positive integer k.
6. Show that if $a \equiv b(\mod n)$ and $b \equiv c(\mod n)$, then $a \equiv c(\mod n)$.
7. State the conditions for proper conics
8. Show that if $a \equiv b(\mod n)$, then $a + c \equiv b + c(\mod n)$.
9. Evaluate $f(2)$ where $f(x) = \log x + x - \cos x$.
10. State the conditions for coincidence and perpendicularity of pair of straight lines.
11. Find the equation of normal to parabola $y^2 = 4ax$ at the point (x_1, y_1).
12. Find the latus rectum, eccentricity and coordinates of foci of $9x^2 + 5y^2 - 30y = 0$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Find the equation of pair of tangents to a conic.
14. Prove that if $ca \equiv cb(\mod n)$ and $\gcd(c, n) = 1$, then $a \equiv b(\mod n)$.
15. State the type of conic $8x^2 - 12xy + 17y^2 + 16x - 12y + 3 = 0$.
16. State and prove Fermat's theorem.
17. Find the chord of contact of the point $(2, 3)$ with respect the conic $2x^2 + 6xy + 4y^2 - 8x + 7 = 0$.
18. Find a real root of the equation $x^3 = 1 - x^2$ on the interval $[0, 1]$ with an accuracy of 10^{-4}.
19. Give the sequence of steps in the regula-falsi method for determining a real root of the equation $f(x) = 0$.
20. Show that the locus of all points the sum of whose distances from two fixed points is constant is an ellipse.

(5 x 5 = 25)

PART C
Answer any 3 (10 marks each)

21. State and prove Wilson's theorem.

22. Use bisection method to find a root correct to three decimal places and lying between 0 and 0.5, of the equation $4e^{-x}\sin x - 1 = 0$.

23. Reduce the equation $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$ to canonical form.

24. Prove that the equation of the chord joining the points $\theta = \theta_1,\ \theta = \theta_2$ on the circle $r = 2a\cos\theta$ is $r\cos(\theta - \theta_1 - \theta_2) = 2a\cos\theta_1\cos\theta_2$.

(10 x 3 = 30)