Reg. No .... Name **23U251**

END SEMESTER EXAMINATION : MARCH 2023

SEMESTER 2 : INTEGRATED M.Sc. PROGRAMME COMPUTER SCIENCE

COURSE : **21UP2CPCMT02 : MATHEMATICS - II - LINEAR ALGEBRA**

(For Regular - 2022 Admission and Improvement / Supplementary - 2021 Admission)

Time : Three Hours **Max. Weightage: 30**

PART A

Answer Any 8 Questions

1. Prove that a0 = 0 for every a ∈ F
2. Define linear independence and linear dependence
3. Evaluate (2+3i)(4+5i).
4. Determine whether the following transformations are linear or not;

    1. T: $R^2 \rightarrow R^2$ defined by T(a,b) = (2a,3b)
    2. T: $R^2 \rightarrow R^2$ defined by T(a,b) = (a+2, b-2)

5. Define LInear Map with an Example
6. Define Eigenvalue and Eigenvector
7. Let T ∈ L(F^3) defined by T(x,y,z) = (2x+y, 5y+3z, 8z).What are the eigen values of T
8. Suppose v ∈ V ,then prove the following;

    1. ‖v‖ =0 if and only if v=0
    2. ‖ λ v‖ = |λ | ‖v‖ for all λ ∈ F

9. Define orthogonal vectors and check the following vectors are orthogonal; (1,-1,0), (2,2,2)
10. Define orthonormal list of vectors

(1 x 8 = 8 Weight)

PART B

Answer Any 6 Questions

11. Prove that every element in a vector space has a unique additive inverses.
12. Check whether the list (1,2,1), (2,1,0),(1,-2,2) is a basis in F^3
13. Suppose V and W are finite dimensional vectorspaces such that dimV < dimW. Then prove that no linear map from V to W is surjective.
14. Find a basis of $P_2(R) \times R^2$
15. Suppose p,q ∈ P(F) and T ∈ L(V) ,then

    1. (pq) (T) =p(T) q(T)
    2. p(T) q(T) = q(T) p(T)

16. Check whether T ∈ L(P^1) defined by T(at+b) = (2a-3b)t +(a-2b) is diagonalizable with respect to the basis 3t+1,t+1 of P^1.Give valuable reason for your answer.

17. Check the following list of vectors are orthonormal list in F^3,
$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$

18. Prove that Every finite dimensional inner product space has an orthonormal basis

(2 x 6 = 12 Weight)

PART C

Answer Any 2 Questions

19. Prove that Every spanning list in a vector space can be reduced to a basis of the vector space and also prove that every linearly independent list of vectors in a finite dimensional vector space can be extended to a basis of the vector space.

20. State and prove the Fundamental theorem of Linear maps

21.
   1. Check whether $T \in L(F^2)$ defined by T(x,y) = (41x+7y, -20x+74y) are diagonalizable either over the standard basis of F^2 or with respect to the basis (1,4),(7,5) of F^2
   2. Check whether $T \in L(P^1)$ defined by T(at+b) = (a+2b)t +(4a+3b) is diagonalizable either over the standard basis of P^1 or with respect to the basis -t+1,5t+10 of P^1. Give valuable reason for your answers.

22. Find an Orthonormal basis of $P_2(R)$,where the innerproduct is given by $< p,q >$ $= \int_{-1}^{1} p\left(x\right)q\left(x\right)\,dx$

(5 x 2 = 10 Weight)