B. Sc. DEGREE END SEMESTER EXAMINATION – OCTOBER 2022

SEMESTER – 3: MATHEMATICS (COMPLEMENTARY COURSE FOR PHYSICS AND CHEMSITRY)

COURSE: **15U3CPMAT3**, VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

(Common for Supplementary 2018 /2017/2016 /2015 Admissions)

Time: Three Hours   Max. Marks: 75

PART A

Each question carries 1 mark. Answer all the questions

1. Define Torsion.
2. Define Binormal vector
3. Define Directional Derivative.
4. Define potential function
5. Solve: $\frac{dy}{dx} + \frac{y}{x} = x^3 - 3$
6. Define Lagrange's differential equation
7. Define a linear differential equation.
8. Give the definition of an exact differential equation
9. Find the focus of the parabola $y^2 = 10x$.
10. Find the centre and foci of an ellipse $\frac{(x-7)^2}{4} + \frac{(y-5)^2}{25} = 1$ (1 x 10 = 10)

PART B

Each question carries 2 marks. Answer any eight

11. Find the length of one turn of the helix $\vec{R} = \cos t \, \hat{\imath} + \sin t \, \hat{\jmath} + t\hat{k}$.
12. Find the gradient of the function f(x, y) = y − x at (2,1).
13. Find the Divergence and curl of the vector $\vec{V} = (3x^2 - 3yz)\hat{\imath} + (3y^2 - 3zx)\hat{\jmath} + (3z^2 - 3xy)\hat{k}$
14. Show that $\vec{F} = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$ is conservative.
15. Find a function $f(x, y, z)$ such that $\vec{F} = \nabla f$ where $\vec{F} = 2x\hat{\imath} + 3y\hat{\jmath} + 4z\hat{k}$.
16. Solve: $(3x^2 + 6xy^2)dx + (6x^2y + 4y^3)dy = 0$
17. Draw the graph of the polar coordinates satisfying 1 ≤ r ≤ 2 and 0 ≤ θ ≤ π/2.
18. Find the eccentricity and the directrices of the ellipse $\frac{x^2}{7} + \frac{y^2}{16} = 1$.
19. Solve $\frac{dy}{dx} = \frac{2x + 3y}{3x + 2y}$
20. Find the slope and intercept of r cos θ = r sin θ by finding its equivalent Cartesian equation.

(2 x 8 = 16)

PART C

Each question carries 5 marks. Answer any five.

21. Show that the curvature of a circle of radius a is $\frac{1}{a}$

22. Find the unit tangent vector of the curve $\overrightarrow{r(t)} = 3\cos t\,\hat{\imath} + 3\sin t\,\hat{\jmath} + t^2\hat{k}$.

23. Find the Binormal vector $\overrightarrow{r(t)} = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + (-1)\hat{k}$ at $t = \frac{\pi}{4}$.

24. Use Green's theorem in a plane to evaluate the integral $\oint_C (y - \sin x)\,dx + (\cos x)dy$ where C is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$

25. Solve: $y + px = x^4p^2$

26. Solve: $(x^2 + y^2 - a^2)xdx + (x^2 - y^2 - b^2)ydy = 0$.

27. Find the eccentricity, foci and directrices of the ellipse $6x^2 + 9y^2 = 54$

(5 x 5 = 25)

PART D

Each question carries 12 marks. Answer any two.

28. Verify Divergence theorem for $\vec{F} = x^2\hat{\imath} + z\hat{\jmath} + yz\hat{k}$ taken over the cube bounded by

$x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

29. Solve: $p = \tan\left(x - \frac{p}{1+p^2}\right)$

30. Find the centre, foci, eccentricity and asymptotes of the conic

$5x^2 - 4y^2 + 20x + 8y - 4 = 0$:

31. Identify the conic $x^2 - y^2 - 2x + 4y = 4$. Find its center, foci, vertices and asymptotes

(12 x 2 = 24)