Reg. No .... Name **22U354**

B. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER 3 : MATHEMATICS (CORE COURSE) FOR B Sc COMPUTER APPLICATION

COURSE : **19U3CRCMT4 : VECTOR CALCULUS, TRIGONOMETRY AND MATRICES**

(For Regular - 2021 Admission and Improvement / Supplementary - 2020 / 2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. If A and B are vector functions then $\nabla\times(A+B) = \nabla\times A + \nabla\times B$
2. Show that $\vec{F} = \left(2x^2+4y^2-3z^2\right)\boldsymbol{i} + \left(8xy-y^2+2z^2\right)\boldsymbol{j} + \left(4yz+z^2-6zx\right)\boldsymbol{k}$ is irrotational.
3. If V is a vector function, prove that $\nabla\cdot(\nabla\times V)=0$
4. If **A**(t) =(3t^2-2t)**i**+(6t-4)**j**+4t**k**, evaluate $\int_2^3 A\left(t\right)dt$.
5. State Guass's divergence theorem.
6. If **F**=3xy**i**-y^2**j**, evaluate $\int_C \mathrm{F}.\,\mathrm{d}\,\mathrm{r}$,where**C** is the arc of the parabola y=2x^2 from (0,0) to (1,2).
7. Separate into real and imaginary parts the expression sech(x+iy).
8. Prove that $tanh(x-y)$ = $\dfrac{tanhx-tanhy}{1-tanhxtanhy}$.
9. Prove that $\sin h\ 2x = \frac{2\ \tan h\ x}{1-\tan h^2\ x}$
10. Find the characteristic equation for the matrix $\begin{bmatrix}1&1&2\\1&2&1\\2&1&1\end{bmatrix}$
11. Find the rank of $\begin{bmatrix}1&2&3&4\\-2&0&5&7\end{bmatrix}$
12. Find the eigen values of the matrix $\begin{bmatrix}1&-1&0\\2&3&2\\1&1&2\end{bmatrix}$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Find the directional derivative of the function $x2(y+z)$ at (1,1, 0) in the direction of the line joining the origin to the point.
14. Find a unit vector normal to the surface $xy^3z^2 = 4$ at the point (-1,-1,2).
15. If **r**(t)=5t^2**i**+t**j**-t^3**k**, prove that $\int_1^2 \left(\mathrm{r}\times\frac{\mathrm{d}^2\mathrm{r}}{\mathrm{dt}^2}\right)\mathrm{d}\,\mathrm{t}$=-14**i**+75**j**-15**k**.

16. The acceleration of a particle at time t is given by
a=18 cos 3t**i**-8 sin 2t**j**+6t**k**.If the velocity **v** and displacement **r** be zero at t=0,find **v** and **r** at any point of t.

17. If $\tan\ \frac{\theta}{2} = \tan h\ \frac{u}{2},\ show\ that\ \sin h\ u = \tan\ \theta$

18. Factorise x^7-1 into real factors.

19. Solve the system of equations by matrix method.
x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6.

20. Find the inverse of A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ by Guass Jordan method.

(5 x 5 = 25)

PART C
Answer any 3 (10 marks each)

21. If **A** is a vector function and φ is a scalar function,then prove that
a)div(φ**A**) =φ div**A** + (gradφ).**A**
b)curl(φ**A**) =φ curl**A** + (gradφ)x**A**

22. Verify Green's theorem in the plane for $\oint_C\ (xy\ dx +\ x^2\ dy\ )$, where C is the curve enclosing the region bounded by the parabola y=x^2 and the line y=x.

23. Find the sum to infinity of the following series
$c\ \sin\ \alpha + \frac{1}{3}c^3 \sin\ 3\alpha + \frac{1}{5}c^5 \sin\ 5\alpha + \ldots\ldots\ldots\ldots\ldots\ldots$

24. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(10 x 3 = 30)