

B. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022
SEMESTER 3 : STATISTICS (FOR MATHEMATICS AND COMPUTER APPLICATION)

COURSE : 19U3CPSTA3 / 19U3CRCST3 : PROBABILITY DISTRIBUTIONS

(For Regular - 2021 Admission and Improvement /Supplementary - 2020 / 2019 Admissions)

Time : Three Hours

Max. Marks: 75

(Use of Scientific Calculator and Statistical tables are permitted)

PART A

(Each Question carries 1 mark. Maximum marks from this part is 10)

1. If X_1, X_2, \dots, X_{16} is a random sample from a normal distribution with mean 10 and s.d. 4, obtain the distribution of $(\sum X_i)/16$ for $i=1,2,3,\dots,16$.
2. If X_1, X_2, \dots, X_{25} is a random sample from a normal distribution with mean 4 and s.d. 2. Find the standard error of the sample mean?
3. State Bernoulli's law of large numbers.
4. Define a Uniform distribution?
5. A man tosses a fair coin 10 times . Find the probability that he will have atleast 5 heads?
6. Define Rectangular distribution?
7. What is the square of a random variable following t distribution with n degrees of freedom?
8. State Tchebueff's inequality?
9. State multiplication theorem on expectation?
10. If X and y are two independent random variables with standard deviations 3 and 2 respectively, find the variance of $2x-3y$
11. What is the distribution of ratio of two independent random variables following Gamma distribution?
12. If $V(X) = 1$ then find $V(2X + 3)$

PART B

(Each question carries 3 marks. Maximum marks from this part is 15)

13. Show that if X is a non negative random variable such that both $E(X)$ and $E(1/X)$ exists, then $E(1/X) \geq 1/E(X)$
14. What is stratified sampling? What are the advantages of this method of sampling?
15. 2% of hooks manufactured by a firm are found to be defective. Find the probability that a box containing 100 hooks have
 - (i) exactly 4 defectives
 - (ii) More than one defective
16. During a war one ship out of 9 was sunk on an average in making a certain voyage, what was the probability that 3 out of a convoy of 6 ships would arrive safely?

17. What is the relationship between exponential and gamma distributions?
18. Obtain the M.G.F. of the random variable X having p.d.f. $f(x) = x, 0 < x < 1$
 $= 2-x, 1 < x < 2$
 $= 0$ otherwise
19. What are the advantages and disadvantages of Tchebycheff's inequality

PART C

(Each question carries 5 marks. Maximum marks from this part is 20)

20. In a certain factory turning out optical lenses there is a small chance of one out of 1500 for any one lens to be defective. The lenses are supplied in packets of 4500 each. Use Poisson distribution to calculate the approximate number of packets containing one defective lens in a consignment of 20,000 packets
21. Random variable X has the p.d.f. given by $f(x) = 2e^{-2x}, x > 0$ and $= 0$ if $x \leq 0$. Find the M.G.F. Also find the first four moments about the origin?
22. The number of aeroplanes arriving at a certain airport in any 20 minute period follows Poisson distribution with mean 100. Use Tchebycheff's inequality to find a lower bound for the probability that the number of aeroplanes arriving in a given 20 minute period will be between 80 and 120.
23. Define (1) simple random sampling (2) systematic sampling (3) stratified sampling
24. The following results were obtained when 100 batches of seeds were allowed to germinate on a damp filter paper in a laboratory $\beta_1 = 1/15$ $\beta_2 = 89/30$. Determine the binomial distribution and calculate the frequency for $X=6$ when $P > q$
25. State 'Lack of memory' property. Show that Geometric distribution possess the lack of memory property?

PART D

(Each question carries 10 marks. Maximum marks from this part is 30)

26. Find the correlation coefficient between X and Y given that $f(XY) = X+Y$ for $0 < X < 1; 0 < Y < 1$
27. Define normal distribution? Explain the characteristics of normal distribution? Find its mean and variance?
28. i) Define chi-square distribution and state its applications ii) Define 't' distribution and state its assumptions iii) Define 'F' distribution
29. (i) State and prove Lindberg levy form of central limit theorem (ii) A scientist desires to estimate the mean of a population using a sample sufficiently large, such that the probability will be 0.99 that the sample mean will not differ from the population mean by more than 25% of the standard deviation. How large the size of the sample should be?