Reg. No……………………….…………. Name………………………………………**22U321-S**

B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2022

SEMESTER **– 3 : CORE COURSE FOR MATHEMATICS AND COMPUTER APPLICATIONS**

COURSE: **15U3CRMAT3-15U3CRCMT3; CALCULUS**

(Common for Supplementary 2015/2016/2017/2018 Admissions)

Time: Three Hours Max Marks: 75

PART A

Answer all questions. Each question carries 1 mark

1. State Leibnitz theorem.
2. Find the n^{th} derivative of $y = a^{mx}$.
3. What is the relation between the evolute and envelope of a curve
4. State Euler's Mixed Derivative Theorem.
5. Define critical point of a function f(x, y).
6. Evaluate $\int_0^{\frac{\pi}{4}} tanxsec^2x \quad dx$
7. Write surface area formula for revolution about *y* axis.
8. State first derivative test for local extreme values of f(x, y).
9. Evaluate $\int_{-2}^{2} \quad (x^4 – 4x^2 + 6)dx$
10. Find a spherical coordinate equation for the sphere $x^2 + y^2 + (z – 1)^2 = 1$.

(1 x 10 = 10)

PART B

Answer any eight questions. Each question carries 2 mark

11. Find all asymptotes of the curve $y^3 – 6xy^2 + 11x^2y – 6x^3 + x + y = 0$.
12. Determine the points of inflexion of the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$.
13. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (4, -5) if $f(x,y) = x^2 + 3xy+y-1$
14. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r – s$, $y = r + s$
15. Draw a tree diagram for the chain rule for functions of 3 variables
16. Find the area of surface of the region generated by revolving the curve $x = y^3/3$, $0 \le y \le 1$ about x axis.
17. Find the length of the curve $y = x^{3/2}$ from x = 0 to x = 4.
18. Find a spherical co-ordinate equation for the cone $z = \sqrt{x^2 + y^2}$
19. Calculate $\iint_R xy \ dx \, dy$ where R is the region of the circle $x^2 + y^2 = 25, x \geq 0, y \geq 0$.
20. Evaluate the integral by changing in to polar Integral, $\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} dydx$

(2 x 8 = 16)

PART C

Answer any five questions. Each question carries 5 mark

21. Find the points of inflection on the curve $y = \frac{a^2x}{x^2+a^2}$ and show that they lie on a straight line.
22. Find the area of the surface generated by revolving the curve y= $2\sqrt{x}$, $1 \le x \le 2$ about the x axis.
23. Obtain the evolute of the parabola $y^2 = 4ax$.
24. Find the points of inflexion on the curve $y = (\log x)^3$.
25. Find the area of the region enclosed by the parabola $y = 2 - x^2$ $and\ y = -x$
26. Find the centre of curvature of $y^2=4ax$
27. Evaluate the integral $\int_0^1 \int_0^\pi \int_0^{\frac{\pi}{4}} 12\rho\ sin^3\phi\ d\phi d\theta d\rho$

(5 x 5 = 25)

PART D

Answer any two questions. Each question carries 12 mark.

28. a) If $y=e^{\tan^{-1}x}$, prove that $(1+x^2)\ y_{n+1} + 2n(x-1)\ y_n + m(n-1)y_{n-1}=0$

    b) Find the asymptotes of $y^3-6xy^2+11x^2y-6x^3+x+y=0$
29. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.
30. Find the volume of the region D enclosed by the surfaces $Z = x^2 + 3y^2$ and $Z = 8 - x^2 - y^2$
31. a. Find the length of the astroid x + Cos^3t, y = Sin^3t, $0 \le t \le 2 \ge$

    b. The line segment x = 1-y, $0 \le y \le 1$ is revolved about the y axis to generate a cone. Find its lateral surface area. (which excludes the base area)

(12 x 2 = 24)

****